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REDUNDANCY THAT WORKS: HOW K-OUT-OF-N LOGIC PROTECTS SUPPLY CHAINS

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Abstract: *A comprehensive review of the k-out-of-n reliability framework is presented in this paper, and its potential for enhancing robustness in logistics and supply chain systems is demonstrated. Through an examination of a wide range of published research, it is shown that k-out-of-n models have evolved from simple binary formulations to sophisticated multi-state, dependent, and time-dependent structures capable of capturing real-world operational uncertainty. The flexibility of k-out-of-n logic in representing partial functionality, controlled redundancy, and minimum performance thresholds is highlighted, as these features are regarded as essential for modern logistics infrastructures. Applications of this modeling approach in fleet management, automated warehouses, distribution networks, supplier redundancy, and multimodal transportation are reviewed. Numerical examples are provided to illustrate how system-level availability can be quantified using k-out-of-n logic, thereby offering a transparent analytical basis for strategic and operational decisions. It is concluded that k-out-of-n modeling provides a unifying and versatile toolset for the design of supply chains intended to remain resilient despite component-level failures or disruptions.*

Keywords: *reliability, logistics systems, flexibility, efficiency, k-out-of-n systems*

1. INTRODUCTION

The design and operation of logistics systems play a fundamental role in ensuring the efficient movement of goods, information, and resources across supply chains. In both industrial and service environments, a wide range of planning and control tasks must be carried out, including network and vehicle routing, facility layout design, inventory management, resource allocation, workforce scheduling, and the coordination of material-handling processes. These tasks are typically approached from the perspective of cost minimization, throughput maximization, or service-level improvement, and they rely heavily on quantitative models to support decision making in increasingly complex operational contexts.
Despite the strong emphasis traditionally placed on optimization, efficiency, and cost reduction, the reliability of logistics systems has gained growing attention in recent years. Real-world operations are subject to equipment breakdowns, process delays, uncertain demand, workforce variability, and disruptions of various kinds. As a result, the ability of a logistics system to maintain a minimum level of functionality under uncertainty has become a key performance requirement. Many logistics structures can be represented as k-out-of-n systems, in which overall operation is maintained as long as at least k out of n parallel or interdependent elements remain functional. Such configurations naturally arise in fleets of vehicles, sets of picking or sorting lines, redundant suppliers, automated storage systems, transportation modes, and even staffing arrangements. Because the performance of these subsystems depends on meeting minimum operational thresholds, their reliability must be evaluated, monitored, and often optimized.
Given the pervasive presence of k-out-of-n structures across logistics operations, understanding their behavior provides valuable insight into the robustness and resilience of

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entire supply chains. This paper aims to offer a concise overview of k-out-of-n reliability modeling and to highlight its practical relevance for logistics system design. By synthesizing key results from the literature and discussing illustrative examples, the potential of k-out-of-n logic as a decision-support tool is explored. The intention is not to provide an exhaustive technical treatment, but rather to present the essential concepts and demonstrate how k-out-of-n modeling can contribute to better-informed, reliability-aware logistics planning.

As illustrated in Fig. 1, research on k-out-of-n reliability models has been conducted continuously and with increasing intensity over the past two decades.

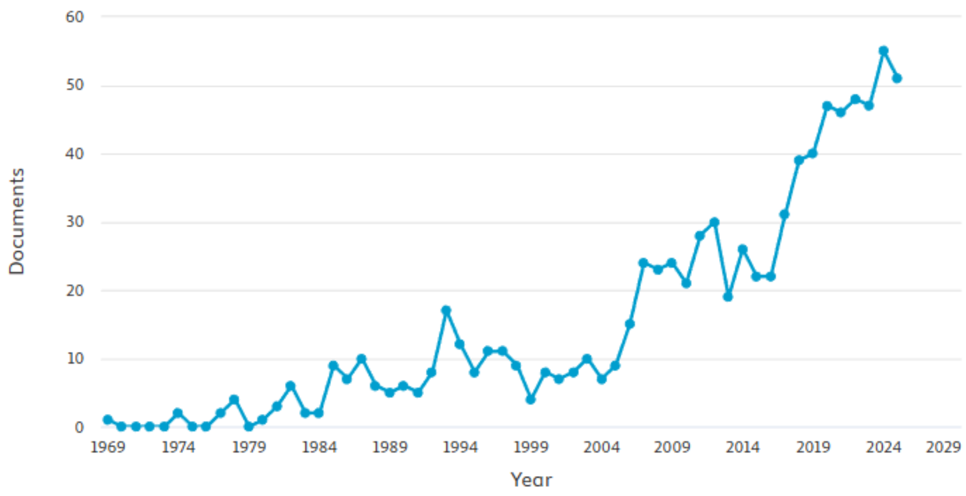

Figure 1. Documents by year in the field of k-out-of-n reliability models. Source: Scopus.

As Fig. 2 depicts, most of the articles were published in journals with production-related topics, but a significant number of the papers were accepted for publication in journals focusing on the application of reliability theory.

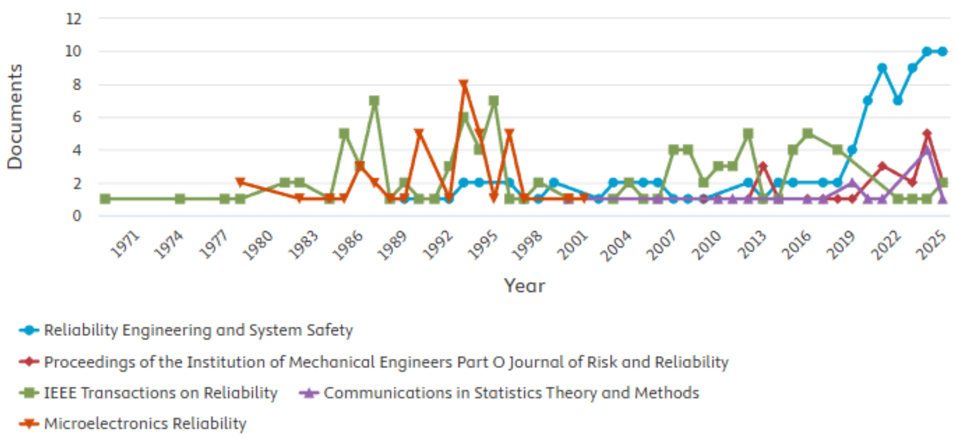

Figure 2. Documents by year per source in the field of k-out-of-n reliability models. Source: Scopus.

The studies on k-out-of-n reliability models can be categorized according to their research areas. Fig. 3 presents the distribution of 2016 articles across ten subject areas. The classification indicates that most of the publications fall within engineering, mathematics and computer science.

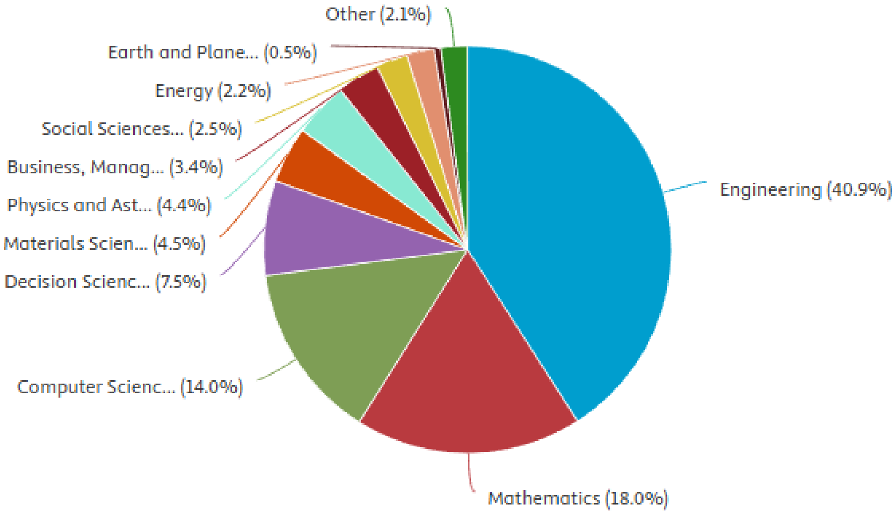

Figure 3. Documents by year per source in the field of k-out-of-n reliability models. Source: Scopus.

Coit et al. (1996) developed a genetic algorithm to optimize redundancy allocation in systems containing series–parallel and k-out-of-n subsystems. Their results show how improving component allocation can significantly enhance the reliability of k-out-of-n structures [1]. Chiang et al. (1981) analyzed the reliability of consecutive-k-out-of-n:F systems and derived recursive formulas tailored to these models. Their work demonstrates how ordered failures influence the operational probability of k-out-of-n-type configurations [2]. Navarro et al. (2008) examined system signatures and showed that k-out-of-n structures play a fundamental role in modeling coherent system lifetimes. Their findings illustrate how complex systems can be represented as mixtures of k-out-of-n signatures [3]. Chao et al. (1995) provided a comprehensive survey of research on consecutive-k-out-of-n reliability systems. Their review highlights analytical techniques that were specifically designed to evaluate k-out-of-n-based configurations [4]. Li et al. (2008) evaluated multi-state weighted k-out-of-n systems by extending classical binary k-out-of-n formulations. Their work demonstrates how varying component performance levels affect the system's ability to satisfy the k-out-of-n requirement [5].

Wu et al. (1994) proposed an efficient algorithm for computing the reliability of weighted k-out-of-n systems. Their approach simplifies reliability evaluation when components contribute unequally toward meeting the k condition [6]. Navarro et al. (2007) developed reliability bounds for coherent systems using underlying k-out-of-n signatures. Their analysis clarifies how dependencies between components influence the effective behavior of k-out-of-n models [7]. Coit et al. (2000) studied redundancy allocation in systems composed of multiple k-out-of-n subsystems. Their optimization framework highlights how the performance of each k-out-of-n block affects overall system reliability [8]. Kuo et al. (2012)

reviewed modern importance measures and examined their applications within k-out-of-n reliability structures. Their results help identify the components that most strongly influence the success probability of k-based configurations [9]. Barlow et al. (1984) developed a simple and efficient algorithm for calculating the reliability of k-out-of-n:G systems. Their work provides a practical way to evaluate systems in which functionality depends on meeting a minimum k threshold [10].

Liu et al. (1998) investigated load-sharing k-out-of-n systems with non-identical component lifetimes. Their results show how increasing stress on surviving units affects the reliability of k-out-of-n configurations [11]. Doostparast et al. (2014) developed a reliability-based preventive maintenance strategy for coherent systems containing k-out-of-n structures. Their approach demonstrates how optimized maintenance can improve the long-term availability of k-out-of-n subsystems [12]. Xie et al. (2004) proposed a load–strength interference model tailored to k-out-of-n systems under dependent failure modes. Their method highlights how shared stresses influence the probability that at least k components remain functional [13]. Shanthikumar et al. (1982) designed a recursive algorithm for computing the reliability of consecutive-k-out-of-n:F systems. Their findings provide an efficient means of evaluating ordered-failure variants of k-out-of-n structures [14]. Eryilmaz et al. (2010) reviewed recent developments in the analysis of consecutive-k-out-of-n reliability models. Their survey places these models within the broader class of k-out-of-n-type systems and highlights key computational advances [15]. She et al. (1992) developed closed-form reliability expressions for warm-standby k-out-of-n systems. Their model unifies warm, cold, and hot standby mechanisms under a generalized k-out-of-n framework [16]. Cai et al. (1991) examined possibilistic reliability behavior in systems that include k-out-of-n-type configurations. Their work shows how uncertainty in component failure impacts the evaluation of systems where at least k units must operate [17]. Zhao et al. (2018) analyzed multi-state k-out-of-n:G systems where performance can be shared over a common bus. Their study demonstrates how performance redistribution among components affects the system's ability to meet the k-out-of-n requirement [18]. Amari et al. (2012) investigated the reliability characteristics of warm-standby k-out-of-n systems. Their results offer analytical expressions that quantify how standby mechanisms influence the survival probability of k-out-of-n configurations [19].

Kim et al. (2011) introduced an extended reliability block diagram capable of modeling k-out-of-n gates. Their approach offers a practical diagrammatic tool for analyzing systems governed by k-out-of-n operational rules [20]. Nakagawa et al. (1977) developed a heuristic method for optimally allocating reliability in systems that may include k-out-of-n subsystems. Their method assigns resources in a way that strengthens system performance relative to the k threshold [21]. Xing et al. (2012) studied phased-mission k-out-of-n systems with imperfect fault coverage. Their analysis demonstrates how different mission phases alter the likelihood of maintaining the required k functioning components [22]. Eryilmaz et al. (2009) analyzed consecutive-k-out-of-n systems with dependent component lifetimes. Their results help quantify how statistical dependence influences achieving the k-out-of-n functional condition [23]. Fu et al. (1987) examined reliability in large consecutive-k-out-of-n:F systems under Markov-dependent failures. Their findings show how temporal dependence among failures affects the probability of meeting the required k operational units [24].

Wang et al. (2017) developed a reliability model for modular multilevel converters incorporating maintenance effects, often modeled with k-out-of-n redundancy. Their results demonstrate how preventive maintenance improves the probability of meeting k-unit operational requirements [25]. Wen et al. (2016) proposed an uncertainty-based reliability analysis framework applicable to systems including k-out-of-n structures. Their method accounts for incomplete information when evaluating the probability of having k functioning units [26]. Ding et al. (2010) provided an approximation framework for multi-state weighted k-out-of-n systems. Their approach reduces computational complexity while preserving accuracy in assessing whether weighted contributions satisfy the k requirement [27]. Chen et al. (2005) studied two-stage weighted k-out-of-n systems with components in common. Their results highlight how shared components influence the probability that each stage satisfies its respective k requirement [28]. Myers et al. (2007) investigated imperfect fault coverage in k-out-of-n:G system structures. Their findings show that even slight reductions in coverage can significantly lower the probability of maintaining k functioning components [29]. Lambiris et al. (1985) derived exact reliability formulas for linear and circular consecutive-k-out-of-n:F systems. Their contributions clarify how system geometry affects achieving the k-out-of-n operational condition [30]. Li et al. (2019) evaluated reliability in LCC and hybrid DC grids, which often utilize k-out-of-n redundancy principles. Their model quantifies how different redundancy allocations impact the probability of meeting k functional units in power grids [31]. Dui et al. (2021) modeled mission reliability for UAV swarms as consecutive-k-out-of-n structures. Their optimization approach identifies swarm sizes that maximize the probability of satisfying the k-unit functional requirement [32]. Li et al. (2007) analyzed δ-shock models that generalize several reliability structures, including k-out-of-n configurations. Their results help evaluate how shock effects influence the likelihood of preserving at least k operational units [33]. Yam et al. (2003) examined repairable circular consecutive-k-out-of-n:F systems with prioritized repairs. Their findings show how repair strategies impact the probability of maintaining k functioning elements in circular arrangements [34]. Feng et al. (2022) studied phased-mission UAV swarm reliability modeled as a k-out-of-n structure. Their work identifies optimal swarm sizes that preserve mission capability by maintaining k operational UAVs [35]. Cui et al. (2018) analyzed k-out-of-n:F balanced systems divided into multiple sectors. Their model describes how sector-level failures influence the system's ability to satisfy the k-out-of-n condition [36]. Eryilmaz et al. (2014) applied copula-based modeling to weighted k-out-of-n systems with dependent components. Their approach quantifies how dependency structures affect meeting the weighted k requirement [37]. Zuo et al. (2000) developed reliability evaluation methods for combined k-out-of-n:F and consecutive-k-out-of-n:F structures. Their algorithms provide efficient tools for analyzing systems that integrate multiple forms of k-out-of-n logic [38].

The surveyed literature demonstrates that the k-out-of-n reliability framework has evolved into one of the most versatile and analytically rich structures in reliability engineering. Across decades of research, authors have extended the classical binary k-out-of-n formulation to encompass weighted, multi-state, dependent, phased-mission, repairable, and load-sharing variants. These developments collectively reveal that the k-out-of-n concept is far more than a simple operational threshold: it functions as a unifying mechanism that explains how partial functionality, redundancy, and structural constraints influence system performance. The studies also highlight that k-out-of-n systems serve as fundamental building blocks for modeling coherent systems, appearing naturally as subsystems in complex networks, power systems, UAV swarms, logistic infrastructures, and manufacturing processes.

A recurring theme in the literature is the tension between analytical tractability and model realism. While early work focused on closed-form solutions for independent and identical components, more recent contributions have incorporated dependencies, non-identical components, and dynamic behaviors using advanced probabilistic tools such as copulas, generating functions, and system signatures. These methodological expansions have significantly improved the accuracy with which k-out-of-n structures can represent real-world systems, particularly in environments where component interactions or mission phases play a critical role. At the same time, computational efficiency remains a central concern, motivating the development of recursive algorithms, heuristic optimizers, and approximation methods that make the evaluation of large-scale k-out-of-n systems feasible.

The literature further reveals that k-out-of-n models are indispensable for practical decision-making in engineering and logistics. They provide a clear framework for evaluating redundancy allocation, optimizing maintenance schedules, estimating mission success probabilities, and analyzing the robustness of subsystems within larger architectures. The adaptability of the k-out-of-n approach allows practitioners to capture minimum-performance requirements in a mathematically coherent way, making it suitable for applications ranging from fleet management to automated warehouses, energy systems, and communication networks.

In summary, the body of research confirms that the k-out-of-n paradigm offers an effective balance between conceptual simplicity and modeling flexibility. Its continuous development ensures its relevance for both theoretical reliability studies and practical engineering applications. As systems become increasingly complex and interconnected, the k-out-of-n framework will remain a cornerstone for understanding and enhancing reliability in settings where partial functionality and redundancy are essential for sustained operation.
The remainder of this article is structured as follows: Section 2 presents the k-out-of-n logic. Section 3 summarizes the typical potential applications fields in logistics, while Section 4 details some numerical studies to demonstrate the potential in redundancys. Section 5 discusses the implications for industrial practice, followed by concluding remarks.

2. K-OUT-OF-N LOGIC

The k-out-of-n system model begins with the assumption that we have a system composed of n individual components, each of which can be in one of two possible states: working or failed. The system itself is considered functional if the total number of working components reaches or exceeds a prescribed threshold k. Thus, the model does not require that all components operate perfectly, but rather that a sufficient number remain operational. This threshold must satisfy the natural condition $1 \leq k \leq n$. When $k = 1$, the system functions as long as at least one component works, which corresponds to a fully parallel structure. When $k = n$, the system only functions if every component is operational, and this reduces to a classical series system. In the more general and practically more interesting case where $1 < k < n$, the system's behavior lies between these two extremes and allows for controlled redundancy [39].

Since each component is assumed to have exactly two states, (working (denoted as 1) or failed (denoted as 0)), the system state is also binary. The system is said to be in state 1 (working) precisely when the number of components currently in state 1 is at least k. If we let X represent the number of components that are currently working, then the defining condition of a k-out-of-n system can be expressed as follows:

$$X \geq k \ \leftrightarrow System\, works. \tag{1}$$

To proceed with a probabilistic interpretation, it is common to assume that each component operates independently and has the same probability of being in a working state. Let p denote the probability that a single component is working and $q = 1 - p$ the probability that it is failed. Under the independence and identical-component assumptions, the random variable X, representing the number of working components, follows a binomial distribution with parameters nnn and p. Consequently, the probability that exactly i components out of the total n are working is given by:

$$P(X = i) = \binom{n}{i} \cdot p^i \cdot (1-p)^{n-i} \tag{2}$$

This provides the foundation for expressing the probability that the entire system is functioning. Since the system works whenever the number of working components is at least k, the system reliability can be written as the sum of the probabilities of all states in which X takes a value from k up to n:

$$R_{system} = P(X \geq k) = \sum_{i=k}^{n} \binom{n}{i} \cdot p^i \cdot (1-p)^{n-i}, \tag{3}$$

where $\binom{n}{i}$ defines how many ways we can choose which i components are working, p^i is the probability those i chosen components are working, and $(1-p)^{n-i}$ is the probability that the remaining $n - i$ compopnents are not working.

This expression is the fundamental reliability formula for k-out-of-n models under binomial assumptions. It captures the essential idea that system reliability increases with both the reliability of individual components (the value of p) and the degree of built-in redundancy (having a larger number of components relative to the threshold k). The generality and simplicity of this formulation make k-out-of-n models extremely useful in analyzing technical and logistical systems where a minimum operational capacity must be maintained despite possible component failures [39].

Often, reliability depends on time, because components age and fail. Again assuming identical, independent components, the system reliability as a function of time is:

$$R_{system}(t) = \sum_{i=k}^{n} \binom{n}{i} \cdot [R(t)]^i \cdot [1 - R(t)]^{n-i}. \tag{4}$$

This is just equation (3) with p replaced by $R(t)$. We can interpret this as at time t, each component is "up" with probability $R(t)$. This makes k-out-of-n systems very natural in reliability engineering, because once you know the reliability curve $R(t)$for a single component, you can build the system reliability from it.

A particularly important and widely used special case in reliability engineering arises when each component in the system is assumed to have an exponentially distributed lifetime. The exponential model is attractive not only because of its mathematical simplicity but also because it embodies the assumption of a constant failure rate, meaning that the likelihood of a component failing in the next instant does not depend on its age. This assumption is often realistic for electronic components, mechanical units operating in stable environments, or systems in which early-life and wear-out effects are negligible. When each component fails independently and shares the same failure rate $\lambda > 0$, the lifetime T of a single component follows an exponential distribution with the reliability function:

$$R(t) = P(T > t) = e^{-\lambda \cdot t}. \tag{5}$$

This expression states that the probability of a component surviving until time ttt decreases exponentially as time increases. Once this single-component reliability function is introduced into the k-out-of-n framework, the entire system's reliability can be computed by substituting $R(t) = e^{-\lambda \cdot t}$ into the general time-dependent reliability formula. Because the components are identical and independent, the probability that exactly i out of the nnn components are functioning at time t becomes

$$P(X(t) = i) = \binom{n}{i} \cdot (e^{-\lambda \cdot t})^i \cdot (1 - e^{-\lambda \cdot t})^{n-i}. \tag{6}$$

The system is operational whenever at least k components remain functioning, so its reliability at time t is obtained by summing the probabilities of all states in which this condition is met. The resulting expression for the system reliability is

$$R_{system}(t) = \sum_{i=k}^{n} \binom{n}{i} \cdot (e^{-\lambda \cdot t})^i \cdot \left[1 - (e^{-\lambda \cdot t})^i\right]^{n-i}. \tag{7}$$

This formula describes how the system reliability decays as time progresses, starting from a value of 1 at time zero and gradually approaching zero as $t \to \infty$. The functional form is governed entirely by the exponential decay of the individual components. Systems with more redundancy tend to exhibit significantly slower reliability degradation because many components must fail before the system becomes inoperative. Conversely, if k is close to n, even a small number of failures can cause system shutdown, and the reliability curve falls more rapidly.

The exponential-lifetime special case also provides important structural insights. Because the hazard rate is constant, there is no memory effect: the future behavior of each component is independent of how long it has already operated. This property makes the exponential distribution unique among lifetime models and contributes to the mathematical tractability of the k-out-of-n analysis. It also allows engineers to compute system mean time to failure, compare different redundancy levels analytically, and determine how component failure rates influence the overall reliability curve. In summary, the exponential special case forms a cornerstone of reliability theory and provides a clear, interpretable foundation for studying the dynamic behavior of k-out-of-n systems under assumptions of independent, memoryless component failures.

Within the general framework of k-out-of-n reliability models, two classical system configurations emerge naturally as limiting cases: series systems and parallel systems. These two structures represent the simplest and most frequently encountered arrangements in reliability engineering, and they provide useful conceptual anchors for understanding how the choice of k shapes a system's robustness. Both systems can be characterized entirely through the k-out-of-n definition, and doing so reveals their underlying similarity despite the drastic differences in their behavior.

A series system corresponds to the special case in which the system requires all of its components to be functioning simultaneously. In the language of the k-out-of-n model, this is the configuration with $k = n$. The defining characteristic of a series system is that the failure of any single component causes the entire system to fail. If each component has a reliability function $R(t)$, representing the probability that it is still operating at time ttt, then the probability that all nnn components survive to that moment is simply the product of their survival probabilities. Under the standard assumption of independence, the system reliability becomes

$$R_{series}(t) = [R(t)]^n. \quad (8)$$

This formula encapsulates the fragility of series systems: even when each individual component is highly reliable, the combined system reliability may deteriorate rapidly as the number of components increases. For this reason, purely series configurations are suitable only when components are extremely reliable or when system failure carries relatively low consequences. The k-out-of-n perspective shows that a series system has no redundancy at all; every component is essential.

In contrast, a parallel system represents the opposite extreme. Here the system remains functional as long as at least one of its components is still working. This corresponds to the case $k = 1$ in the k-out-of-n model. Because failure occurs only when *all* components have failed, the probability of system failure at time ttt is the probability that none of the components is still functioning. Using the independence assumption, the probability that an individual component has failed by time t is $1 - R(t)$ and thus the probability that all nnn components have failed is $[1 - R(t)]^n$. Consequently, the reliability of a parallel system is given by

$$R_{parallel}(t) = 1 - [1 - R(t)]^n. \quad (9)$$

This expression highlights the robustness inherent in parallel configurations. Because the failure of one component does not compromise system performance as long as others remain operational, parallel systems are well-suited to critical applications where redundancy is desirable or essential. Even when individual components are only moderately reliable, the system as a whole can achieve extremely high reliability through parallel redundancy. In the language of the k-out-of-n model, the system has maximal redundancy because it needs only one successful component to function.

Together, the series and parallel systems illustrate the two extremes of the k-out-of-n framework. When $k = n$, the system is highly vulnerable, collapsing upon the failure of a single component; when $k = 1$, the system is remarkably robust, requiring total component failure before it ceases to function. All other k-out-of-n configurations fall between these extremes, allowing designers to fine-tune redundancy levels according to operational requirements, cost constraints, and acceptable risk. By viewing series and parallel structures through the unified lens of the k-out-of-n model, reliability analysis gains a conceptual coherence that makes it easier to compare, evaluate, and design systems with precisely calibrated fault-tolerance properties.

3. Logistic applications of the k-out-of-n System Concept

In logistics, operational reliability is a critical factor that determines whether goods can move efficiently through fleets, warehouses, and supply chain networks. One powerful way to analyze and improve this reliability is through k-out-of-n system modeling, a method that evaluates how many components of a system must remain operational for the entire process to function. This approach captures the realities of modern logistics, where redundancy, parallel operations, and backup capacity are essential for maintaining performance under uncertainty. The following seven examples illustrate how k-out-of-n logic can be applied across different logistic domains to enhance resilience, optimize resources, and ensure consistent service levels.

3.1. Fleet reliability

A logistics company often operates a fleet of trucks where daily delivery capacity must reach a minimum threshold. In a k-out-of-n setting, this means at least k trucks must be operational out of the total n available. Fleet managers account for vehicle breakdowns, scheduled maintenance, and unexpected delays. The model helps estimate the probability that enough vehicles will be available to meet delivery commitments. It also supports decisions about how many backup trucks the company should own or lease. A larger fleet (higher n) increases reliability but raises costs. Using k-out-of-n analysis, companies can determine the optimal trade-off between cost and reliability. For example, if deliveries require at least 15 trucks, but the company owns 20, they can compute the likelihood that 15 trucks will be usable at any time. It also aids in planning seasonal demand peaks when more vehicles are needed. The model highlights the importance of preventative maintenance to keep the operational probability of each truck high. Fleet reliability planning also influences staffing needs for drivers. Companies often simulate different failure rates to plan how many spare vehicles they need. The system supports both strategic long-term planning and daily dispatch operations. Ultimately, k-out-of-n logic ensures that a logistics fleet maintains consistent service levels even with individual truck failures.

3.2. Warehouse Picking or Sorting Systems

Modern warehouses rely heavily on automation, with multiple picking robots or conveyor lines working in parallel. A k-out-of-n system helps determine how many of these units must remain functional to maintain throughput. For example, if a warehouse has 10 picking robots but only needs 7 to meet demand, it operates as a 7-out-of-10 system. This analysis helps determine expected downtime and optimize maintenance cycles. If too many units fail simultaneously, the warehouse may be unable to process orders at required speeds. Managers use reliability assessments to justify investments in redundancy and improved maintenance procedures. The model can also forecast bottlenecks under peak loads. By understanding the failure probability of each unit, warehouse operators can estimate their overall system availability. K-out-of-n modeling supports decision-making about when to expand capacity. It also helps simulate worst-case scenarios, such as power outages or mechanical failures. Predictive maintenance strategies are often based on this type of reliability analysis. Understanding required operational capacity ensures consistent shipping performance. In highly automated centers, these calculations directly affect customer satisfaction through faster and more reliable order fulfillment.

3.3. Supplier Redundancy in the Supply Chain

Modern warehouses rely heavily on automation, with multiple picking robots or conveyor lines working in parallel. A k-out-of-n system helps determine how many of these units must remain functional to maintain throughput. For example, if a warehouse has 10 picking robots but only needs 7 to meet demand, it operates as a 7-out-of-10 system. This analysis helps determine expected downtime and optimize maintenance cycles. If too many units fail simultaneously, the warehouse may be unable to process orders at required speeds. Managers use reliability assessments to justify investments in redundancy and improved maintenance procedures. The model can also forecast bottlenecks under peak loads. By understanding the failure probability of each unit, warehouse operators can estimate their overall system availability. K-out-of-n modeling supports decision-making about when to expand capacity. It also helps simulate worst-case scenarios, such as power outages or mechanical failures. Predictive maintenance strategies are often based on this type of reliability analysis. Understanding required operational capacity ensures consistent shipping performance. In highly automated centers, these calculations directly affect customer satisfaction through faster and more reliable order fulfillment.

3.4. Availability of Distribution Hubs

Large logistics networks include multiple distribution centers or hubs. In a k-out-of-n model, the network remains functional as long as at least k hubs are available. This helps analyze vulnerability to closures caused by weather, labor shortages, or equipment failure. For example, if a company operates 8 hubs but needs only 5 to maintain routing capacity, it functions as a 5-out-of-8 system. Reliability modeling supports decisions about where to build new hubs and how to position stock. Companies can simulate disruptions and evaluate how resilient the network is. This analysis highlights hubs that are critical and must have higher redundancy or protection. The model also helps plan rerouting strategies in case of outages. It is used to evaluate the economic benefits of adding more distribution centers. Managers can quantify how hub failures affect transit times and delivery commitments. The method improves long-term strategic planning and risk mitigation. Overall, k-out-of-n analysis ensures continuity of operations across complex shipping networks.

3.5. E-commerce Order Processing Systems

E-commerce companies rely on multiple parallel systems for order processing, packaging, and server operations. A k-out-of-n model applies when at least k out of n machines, servers, or packaging lines must function to meet daily order volume. This allows companies to design systems that remain robust even under partial failures. For example, if only 12 packaging lines are needed but 15 are available, the system remains operational unless more than 3 lines fail. This helps maintain customer service levels, especially during peak seasons. Reliability modeling informs infrastructure decisions such as adding more processing lines or increasing maintenance teams. Companies use these models to avoid delivery delays caused by technical failures. It also assists in predicting performance under Black Friday or holiday surge conditions. The model supports capacity planning and makes downtime risks more predictable. It highlights where failures would have the most effect on throughput. Redundant digital systems and servers also follow this logic. Ultimately, k-out-of-n reliability ensures fast, uninterrupted order processing.

3.6. Transportation Route Redundancy (Multimodal Logistics)

In transportation networks, goods often have multiple possible routes or modes of transport. A k-out-of-n approach means at least k viable routes must remain available to ensure timely delivery. For example, a company may rely on road, rail, and air routes, requiring at least 2 of the 3 to function reliably. This helps evaluate risk from weather disruptions, strikes, or infrastructure failures. Businesses can simulate how route closures affect delivery times and costs. This model guides decisions about diversifying transportation options. It also helps determine whether contingency routes are economically justified. Reliability analysis may highlight over-reliance on a vulnerable mode, such as a single railway line. Companies can plan alternative routing policies based on these insights. The model assists in creating contracts with multiple carriers. It is crucial for industries with strict delivery deadlines such as pharmaceuticals or perishable goods. Overall, it strengthens the robustness and flexibility of transportation planning.

3.7. Automated Warehouse Machine Lines

In transportation networks, goods often have multiple possible routes or modes of transport. A k-out-of-n approach means at least k viable routes must remain available to ensure timely delivery. For example, a company may rely on road, rail, and air routes, requiring at least 2 of the 3 to function reliably. This helps evaluate risk from weather disruptions, strikes, or infrastructure failures. Businesses can simulate how route closures affect delivery times and costs. This model guides decisions about diversifying transportation options. It also helps determine whether contingency routes are economically justified. Reliability analysis may highlight over-reliance on a vulnerable mode, such as a single railway line. Companies can plan alternative routing policies based on these insights. The model assists in creating contracts with multiple carriers. It is crucial for industries with strict delivery deadlines such as pharmaceuticals or perishable goods. Overall, it strengthens the robustness and flexibility of transportation planning.

4. NUMERICAL EXAMPLES

The following numerical examples illustrate how the k-out-of-n reliability model can be applied in different logistical contexts. By examining both a vehicle fleet and an automated warehouse system, we demonstrate how system-level performance depends on the reliability of individual components.

4.1. Fleet Reliability in a k-out-of-n setting

Consider a logistics company that operates a fleet of ten delivery trucks. Daily transportation commitments require that at least seven of these vehicles be operational; if fewer than seven are available, the company cannot fulfill its scheduled deliveries. We assume that each truck is independently operational on a given day with probability $p = 0.9$, reflecting a high but realistic level of mechanical reliability. Let X denote the number of trucks that are in working condition on a particular day. Under the independence assumption, X follows a binomial distribution with parameters $n = 10$ and $p = 0.9$.

The fleet is considered functional whenever $X \geq 7$. Thus, the fleet reliability is given by the binomial tail probability

$$P(X \geq 7) = \sum_{i=7}^{10} \binom{10}{i} \cdot 0.9^i \cdot 0.1^{10-i}. \tag{10}$$

Evaluating these terms yields contributions from four possible favorable states: exactly seven operational trucks, exactly eight, nine, and all ten. Numerically, these probabilities are approximately 0.0574, 0.1937, 0.3874 and 0.3487 respectively. Summing them produces a total fleet reliability of about $P(X \geq 7) \approx 0.9872$.

This result implies that on any given day, the company has roughly a 98.7% chance of meeting its minimum operational requirement of seven functional vehicles. In practice, such a high reliability level indicates that the chosen fleet size and redundancy strategy are more than sufficient to absorb the day-to-day variability in vehicle availability. The example illustrates clearly how the k-out-of-n model transforms individual component reliability into a meaningful system-level performance metric, helping fleet managers assess whether their resources are adequate for dependable service delivery.

4.2. Operational Reliability of Warehouse Sorting Lines: A k-out-of-n Example

Imagine a modern automated warehouse that relies on a set of identical picking lines to process customer orders throughout the day. The facility is equipped with $n = 12$ automated picking lines, but the daily throughput requirements can be met as long as at least $k = 9$ of these lines remain operational. If fewer than nine lines are functioning, the warehouse cannot sustain the order-processing speed needed to meet delivery deadlines. To model the reliability of this system, suppose each picking line operates independently with a probability of $p = 0.92$ of being functional at the start of a given day. This probability reflects factors such as mechanical reliability, operator availability, and scheduled maintenance routines.

Let X denote the number of picking lines that are operational on a particular day. Under the assumption of independence, X follows a binomial distribution with parameters $n = 12$ and $p = 0.92$. The warehouse's operational requirement corresponds to the event $X \geq 9$. Thus, the reliability of the picking system is given by the upper tail of the binomial distribution:

$$P(X \geq 9) = \sum_{i=9}^{12} \binom{12}{i} \cdot 0.92^i \cdot 0.08^{12-i}. \tag{11}$$

Evaluating these terms yields the following approximate probabilities: the probability of exactly nine operational lines is 0.053, ten operational lines occurs with probability 0.183, eleven lines with probability 0.384, and all twelve lines with probability 0.368. Summing these contributions gives $P(X \geq 9) \approx 0.989$.

This result indicates that on an average day, the warehouse has about a 98.9% chance of maintaining sufficient operational capacity to handle its targeted order volume. From a managerial perspective, this reliability level suggests that while the current redundancy is generally adequate, there remains a non-negligible risk of falling short of service requirements. Depending on business priorities, this might prompt consideration of additional backup lines, more robust maintenance schedules, or policies that dynamically redistribute workloads during partial system failures. The example demonstrates how the k-out-of-n reliability framework provides a clear, quantitative basis for evaluating and optimizing operational resilience in automated warehouse environments.

5. Discussion

The findings of this study show that k-out-of-n reliability modeling closely reflects the operational logic of logistics systems. In practice, fleets of vehicles, picking lines in warehouses, supplier bases, transport routes, and labor pools all rely on maintaining a minimum functional capacity rather than full availability. This threshold-based behavior makes k-out-of-n models particularly suitable for capturing how logistics systems continue to operate despite partial failures. The numerical examples highlight how system-level performance can be evaluated through k-out-of-n logic. In both the fleet and warehouse scenarios, operational success depends on meeting a required minimum number of functioning resources, illustrating how redundancy directly contributes to reliability in logistics environments. Such insights help clarify how capacity decisions, maintenance planning, and resource allocation affect overall service levels.

The literature also indicates that many logistics processes naturally decompose into k-out-of-n subsystems, from supplier redundancy to multimodal transport options. Even when logistics systems involve dependencies or shared workloads, extended k-out-of-n models – such as multi-state or load-sharing formulations – remain capable of representing real operational conditions. Overall, the discussion suggests that k-out-of-n modeling provides a practical and intuitive framework for analyzing and improving logistics system resilience. By aligning reliability considerations with everyday operational structures, these models offer valuable support for designing supply chains that remain functional despite disruptions and uncertainty.

References

[1] Coit, D. W. & Smith, A. (1996). Reliability optimization of series-parallel systems using a genetic algorithm. *IEEE Transactions on Reliability*, **45**(2), 254–260, 263. https://doi.org/10.1109/24.510811

[2] Chiang, D. T. & Niu, S.-C. (1981). Reliability of consecutive-k-out-of-n:F system. *IEEE Transactions on Reliability*, **R-30**(1), 87–89. https://doi.org/10.1109/TR.1981.5220981

[3] Navarro, J., Samaniego, F. J., Balakrishnan, N. & Bhattacharya, D. (2008). On the application and extension of system signatures in engineering reliability. *Naval Research Logistics*, **55**(4), 313–327. https://doi.org/10.1002/nav.20285

[4] Chao, M., Fu, J. & Koutras, M. (1995). Survey of reliability studies of consecutive-k-out-of-n:F and related systems. *IEEE Transactions on Reliability*, **44**(1), 120–127. https://doi.org/10.1109/24.376531

[5] Li, W. & Zuo, M. J. (2008). Reliability evaluation of multi-state weighted k-out-of-n systems. *Reliability Engineering and System Safety*, **93**(1), 160–167. https://doi.org/10.1016/j.ress.2006.11.009

[6] Wu, J.-S. & Chen, R.-J. (1994). An algorithm for computing the reliability of weighted-k-out-of-n systems. *IEEE Transactions on Reliability*, **43**(2), 327–328. https://doi.org/10.1109/24.295016

[7] Navarro, J. & Rychlik, T. (2007). Reliability and expectation bounds for coherent systems with exchangeable components. *Journal of Multivariate Analysis*, **98**(1), 102–113. https://doi.org/10.1016/j.jmva.2005.09.003

[8] Coit, D. W. & Liu, J. (2000). System reliability optimization with k-out-of-n subsystems. *International Journal of Reliability, Quality and Safety Engineering*, **7**(2), 129–142. https://doi.org/10.1142/S0218539300000110

[9] Kuo, W. & Zhu, X. (2012). Some recent advances on importance measures in reliability. *IEEE Transactions on Reliability*, **61**(2), 344–360. https://doi.org/10.1109/TR.2012.2194196

[10] Barlow, R. (1984). Computing k-out-of-n system reliability. *IEEE Transactions on Reliability*, **R-33**(4), 322–323. https://doi.org/10.1109/TR.1984.5221843

[11] Liu, H. (1998). Reliability of a load-sharing k-out-of-n:G system: Non-iid components with arbitrary distributions. *IEEE Transactions on Reliability*, **47**(3), 279–284. https://doi.org/10.1109/24.740502

[12] Doostparast, M., Kolahan, F. & Doostparast, M. (2014). A reliability-based approach to optimize preventive maintenance scheduling for coherent systems. *Reliability Engineering and System Safety*, **126**, 98–106. https://doi.org/10.1016/j.ress.2014.01.010

[13] Xie, L., Zhou, J. & Hao, C. (2004). System-level load-strength interference based reliability modeling of k-out-of-n system. *Reliability Engineering and System Safety*, **84**(3), 311–317. https://doi.org/10.1016/j.ress.2003.12.003

[14] Shanthikumar, J. G. (1982). Recursive algorithm to evaluate the reliability of a consecutive-k-out-of-n:F system. *IEEE Transactions on Reliability*, **R-31**(5), 442–443. https://doi.org/10.1109/TR.1982.5221423

[15] Eryilmaz, S. (2010). Review of recent advances in reliability of consecutive k-out-of-n and related systems. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, **224**(3), 225–237. https://doi.org/10.1243/1748006XJRR332

[16] She, J. & Pecht, M. G. (1992). Reliability of a k-out-of-n warm-standby system. *IEEE Transactions on Reliability,* **41**(1), 72–75. https://doi.org/10.1109/24.126674

[17] Cai, K.-Y., Wen, C.-Y. & Zhang, M.-L. (1991). Pseudoist reliability behavior of typical systems with two types of failure. *Fuzzy Sets and Systems*, **43**(1), 17–32. https://doi.org/10.1016/0165-0114(91)90018-L

[18] Zhao, X., Wu, C., Wang, S. & Wang, X. (2018). Reliability analysis of multi-state k-out-of-n:G systems with common bus performance sharing. *Computers & Industrial Engineering*, **124**, 359–369. https://doi.org/10.1016/j.cie.2018.07.034

[19] Amari, S. V., Pham, H. & Misra, R. B. (2012). Reliability characteristics of k-out-of-n warm standby systems. *IEEE Transactions on Reliability*, **61**(4), 1007–1018. https://doi.org/10.1109/TR.2012.2220891

[20] Kim, M. C. (2011). Reliability block diagram with general gates and its application to system reliability analysis. *Annals of Nuclear Energy*, **38**(11), 2456–2461. https://doi.org/10.1016/j.anucene.2011.07.013

[21] Nakagawa, Y. & Nakashima, K. (1977). A heuristic method for determining optimal reliability allocation. *IEEE Transactions on Reliability*, **R-26**(3), 156–161. https://doi.org/10.1109/TR.1977.5220101

[22] Xing, L., Amari, S. V. & Wang, C. (2012). Reliability of k-out-of-n systems with phased-mission requirements and imperfect fault coverage. *Reliability Engineering and System Safety*, **103**, 45–50. https://doi.org/10.1016/j.ress.2012.03.018

[23] Eryilmaz, S. (2009). Reliability properties of consecutive k-out-of-n systems of arbitrarily dependent components. *Reliability Engineering and System Safety*, **94**(2), 350–356. https://doi.org/10.1016/j.ress.2008.03.027

[24] Fu, J. C. & Hu, B. (1987). On reliability of a large consecutive-k-out-of-n:F system with (k−l)-step Markov dependence. *IEEE Transactions on Reliability*, **R-36**(1), 75–77. https://doi.org/10.1109/TR.1987.5222299

[25] Wang, B., Wang, X., Bie, Z., Judge, P. D. & Wang, X. (2017). Reliability model of MMC considering periodic preventive maintenance. *IEEE Transactions on Power Delivery*, **32**(3), 1535–1544. https://doi.org/10.1109/TPWRD.2016.2602888

[26] Wen, M. & Kang, R. (2016). Reliability analysis in uncertain random system. *Fuzzy Optimization and Decision Making*, **15**(4), 491–506. https://doi.org/10.1007/s10700-016-9235-y

[27] Ding, Y., Zuo, M. J., Lisnianski, A. & Li, W. (2010). A framework for reliability approximation of multi-state weighted k-out-of-n systems. *IEEE Transactions on Reliability*, **59**(2), 297–308. https://doi.org/10.1109/TR.2010.2048659

[28] Chen, Y. & Yang, Q. (2005). Reliability of two-stage weighted-k-out-of-n systems with components in common. *IEEE Transactions on Reliability*, **54**(3), 431–440. https://doi.org/10.1109/TR.2005.853274

[29] Myers, A. F. (2007). k-out-of-n:G system reliability with imperfect fault coverage. *IEEE Transactions on Reliability*, **56**(3), 464–473. https://doi.org/10.1109/TR.2007.903229

[30] Lambiris, M. & Papastavridis, S. (1985). Exact reliability formulas for linear and circular consecutive-k-out-of-n:F systems. *IEEE Transactions on Reliability*, **R-34**(2), 124–126. https://doi.org/10.1109/TR.1985.5221969

[31] Li, G., Liang, J., Joseph, T., An, T. & Lu, J. (2019). Feasibility and reliability analysis of LCC DC grids and LCC/VSC hybrid DC grids. *IEEE Access*, **7**, 22445–22456. https://doi.org/10.1109/ACCESS.2019.2898387

[32] Dui, H., Zhang, C., Bai, G. & Chen, L. (2021). Mission reliability modeling of UAV swarm and its structure optimization based on importance measure. *Reliability Engineering and System Safety*, **215**, Article 107879. https://doi.org/10.1016/j.ress.2021.107879

[33] Li, Z. & Zhao, P. (2007). Reliability analysis on the δ-shock model of complex systems. *IEEE Transactions on Reliability*, **56**(2), 340–348. https://doi.org/10.1109/TR.2007.895306

[34] Yam, R. C. M., Zuo, M. J. & Zhang, Y. L. (2003). A method for evaluation of reliability indices for repairable circular consecutive-k-out-of-n:F systems. *Reliability Engineering and System Safety*, **79**(1), 1–9. https://doi.org/10.1016/S0951-8320(02)00204-1

[35] Feng, Q., Liu, M., Dui, H., Ren, Y. & Sun, B. (2022). Importance measure-based phased mission reliability and UAV number optimization for swarm. *Reliability Engineering and System Safety*, **223**, Article 108478. https://doi.org/10.1016/j.ress.2022.108478

[36] Cui, L., Gao, H. & Mo, Y. (2018). Reliability for k-out-of-n:F balanced systems with m sectors. *IISE Transactions*, **50**(5), 381–393. https://doi.org/10.1080/24725854.2017.1397856

[37] Eryilmaz, S. (2014). Multivariate copula-based dynamic reliability modeling with application to weighted-k-out-of-n systems of dependent components. *Structural Safety*, **51**, 23–28. https://doi.org/10.1016/j.strusafe.2014.05.004

[38] Zuo, M. J. (2000). Reliability evaluation of combined k-out-of-n:F, consecutive-k-out-of-n:F, and linear connected-(r, s)-out-of-(m, n):F system structures. *IEEE Transactions on Reliability*, **49**(1), 99–104. https://doi.org/10.1109/24.855542

[39] Ram, M. & Dohi, T. (2019). ystems Engineering: Reliability Analysis Using k-out-of-n Structures. CRC Press, https://doi.org/10.1201/9781351056465