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MATHEMATICAL MODEL OF PERCEPTION DRIVEN SERVICE

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Abstract: The concept of rethinking, reorganizing or reengineering dysfunctional business processes has been approached from many aspects both by professionals and academic researchers. However, service processes do not get as much attention as their importance would deserve. One reason can be the usage of production-oriented methods for service processes, while another might be the complexity and stochasticity of service processes, which cannot be handled by classical BPR techniques. In these processes, it is usually hard to give strict and consistent parameters, indicators and even objective functions. In our research, we offer a unified, heuristic mapping and simulation methodology for process simulation and improvement. In this paper, this problem is analysed from a very new aspect as the observed processes of the system are not considered as an essential argument of the examination. Instead the main elements are the nodes of the system, transactions and transformations. The second important aspect of the methodology is the quality and efficiency perceptions of internal and external users of the processes. Thirdly, not only cost efficiency and lead-times are applied in the objective function, but also the reduction of process entropy.
Keywords: mathematical model, condition system, objective function, cost function, entropy, time function

1. Introduction

Logistic processes have fundamental role in the field of economy, especially in the service sector. The main goals of every service providing economic actors are to decrease the costs connected to logistic and rationalize the logistic processes in a way that improves and not declines the quality of the service. While observing the logistic of different services it can be observed that many processes are similar. In order to rationalize the logistic processes first the common elements of the service processes should be revealed, and the mathematical model should be made accordingly. Such a model will be presented in this article.

The domination of the service sector keeps growing in the world's largest economies as global statistics show in OECD and World Bank databases. Eichengreen and Gupta [1] argue this as well. More than 63% of world GDP is produced by the service sector [2] and this ratio is even greater in countries with higher GDP (more than 75%). This increasing importance has led us to examine recent business process amelioration methods. We also examine their usability and customization for service processes. This topic of business process improvement or re-engineering has a long history in among managers. There is a lot of confusion and debate on this topic. The reengineering topic has not lost its actuality. The concept of reorganizing dysfunctional business processes still exists even in the twenty-first century. Usually this is with new, more sophisticated tools and methodologies. Yet, it is still based on old principals. The narrowing markets, increasing competition and the recent economic crisis all stimulate companies towards continuous rationalization, cost reduction and increased efficiency. All of this is in order to gain some kind of comparative

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advantage, which creates a basis for the development of methodologies of process improvement. We found that these business process amelioration (BPA) methods and techniques are not clearly adoptable to service processes [3]. Despite the large number of BPA technologies and tools, efforts have tended to emphasize manufacturing applications over service operations. By now it has become apparent that the economies of even the most industrialized countries are becoming increasingly dominated by services. However, producing consistently high quality and efficient services has not received as much attention as in manufacturing firms [1]. The differences in the characteristics of manufacturing and services have led many managers to believe that successful BPA methods used in manufacturing are not applicable in service organizations. There is a lot of evidence, though, of using BPA tools tailored clearly for the service sector (see e.g.: [2], [4], [5]). Due to the lack of service standards, there is general success in the services. This makes the customer-focused approach of BPI inherently attractive for a service organisation. Hence, BPI methodologies have been widely disseminated and adopted, especially in the financial services and healthcare areas [8], [9]. The reason is not only the lack of tools, but also the specifications of services. Human intervention is common practice in services, which results in a lot of hidden factors. Thus, the success of BPA in service organizations depends very much on the fit among interdependence. Also, the success depends on the strategy's content and process [3].

2. The generic design of the mathematical model

A mathematical model can be built on the basis of the previously given definitions and relationships. The aim of this model is to provide theoretical background to our future simulation model and on the basis of this mathematical model the required computing model can be prepared. It should be kept in mind during the modelling that perception driven processes (PDP) must be mapped to the model level. In-model changes shall be reflected to the observed world as if it would take place in real world. This requirement is illustrated in Figure 1. below.

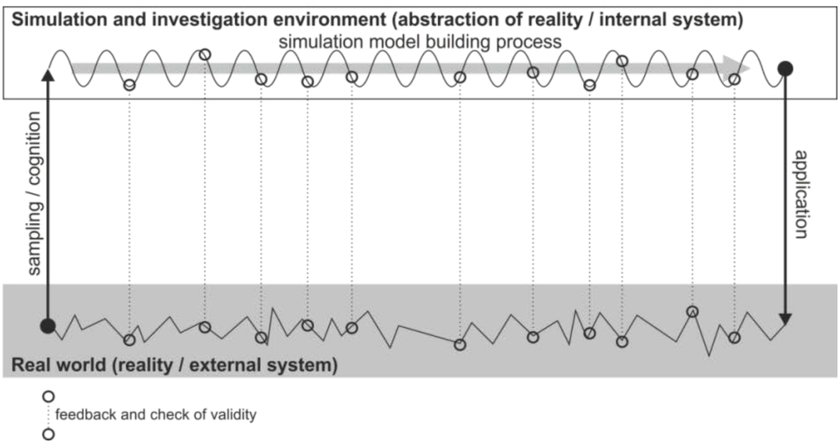

Figure 1. Modelling requirement

However, a general model is created at this stage for each of the above-mentioned processes. It is not worth detailing the model even deeper since the point may be lost. Quantities in the system are the expected values of the random variables. Accordingly, each quantity has a probability distribution whose definition requires empirical studies and schemes may be different for each element.

During this examination of the system, nodes along with their data transactions and possible inter-nodal fluid flows are to be given. These will be given after our future empirical observations. The aim of the model is to develop a method which can provide input fluids (fluids on entry points) are able to reach output fluids (exit points) in the required quality and quantity. This should be implemented optimally. Optimality will be achieved by using the objective function [10, 11].

3. The mathematical model

Based on the concepts of the model, known and unknown variables should be given. In addition, conditionality should be built based on relationships between certain elements of the framework. For the final examination of the system, the objective function is presented in this paper. So as our model consists of two main parts:

- conditionality,
- objective function.

3.1. Known data.

Let

n_p be the number of INPUT nodes,
n_o be the number of OUTPUT nodes, and
n_k be the number of interstitial but external nodes,
n_b be the number of internal nodes of system.

Let

n be the total number of nodes of the system,
n_b be the number of interstitial but external nodes of the system.
Every node is represented by an $i \in \{1, \dots, n\}$ value. $n = n_p + n_k + n_o + n_b$
If $i \in \{1, \dots, n_p\}$ then input node,
If $i \in \{n_i + 1, \dots, n_p + n_k\}$ then interstitial but external node,
If $i \in \{n_p + n_k + 1, \dots, n_p + n_k + n_o\}$ then output node,
If $i \in \{n_p + n_k + n_o + 1, \dots, n\}$ then internal node in the system is considered.

Let

m be the number of possible fluids. Every fluid is represented by a $f \in \{1, \dots, m\}$ value.

Let

F_i^I be the volume of ith input node.
F_i^O be the required volume of ith output node.

Let

$F^I = \{F_i^I\}$ be the set of input nodes sorted by number.

$F^O = \{F_i^O\}$ be the set of output nodes sorted by number.

Let
D^I be the weight of a D node appeared on the ith input node (weight is considered in an abstracted term)
D^O be the weight of a D node required by ith output node (weight is considered in an abstracted term)

Let
$D^I = \{D_i^I\}$ be the set of input fluids arranged by node sequence numbers
$D^O = \{D_i^O\}$ be the set of output fluids arranged by node sequence numbers.

There are some important conclusions of these assumptions. There are special nodes either on input or output side. Input nodes launch a fluid into the examined system through the particular fluid flow and they are unable to accept fluids.

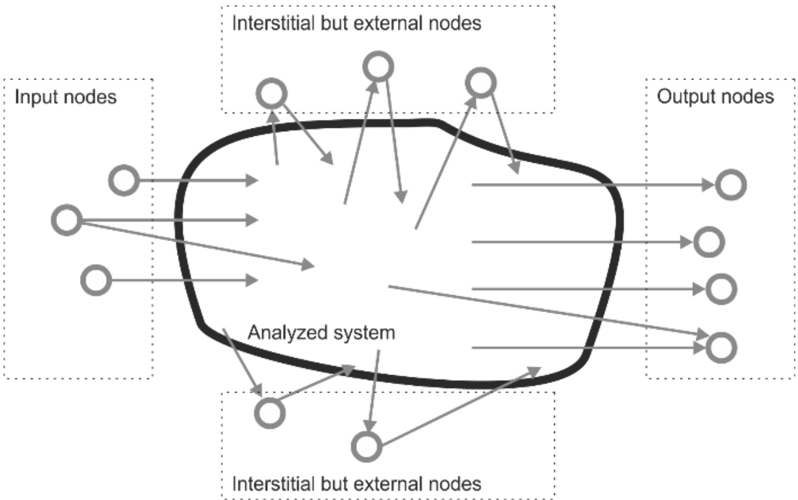

Figure 2. Representation of nodal flow model

Expected fluids will appear on output nodes (in appropriate quantity and quality). Here, the expectations are displayed via node related attributes. Output nodes do not emit fluids. Interstitial, but external nodes are those that are actually users intervening in the system to complete the entire process. They can emit and accept fluids.

The set of these three kinds of nodes is equal to the previously defined set of inter-users. Inter-users can be considered as the same nodes as the internal elements (nodes) of the system. Thus they can be featured by same functions, properties, transformations.

These remarks led us to conclude that a dual system is obtained. The primal system is the examined one while the dual is the independent (possibly dependent) one with external nodes.

It might be considered during our investigation as a joint connection of two of the same fluid flow system.

However, all input-output, interstitial and external nodes can be considered as the same client. In our model, they are considered to be separate nodes for simplicity. This simplification does not violate the principles raised in the previous paragraph.

3.2. Determination of model inputs. Our system is basically built of nodes and fluid flows [12, 13, 14]. Every node is assumed to be an agent (or a computational object). Each node is structured as a set of attributes and transactions. The structure of i^{th} node consists of data and transactions.

3.2.1. Data.

Let

$p_{max_i}(f)$ be the maximal capacity of i^{th} node in relation to fluid *f*. If this value is 0, then *f* fluid cannot flow through this node.

$h_i(f)$ be i^{th} node the compulsory node of fluid *f*. If this value is 0, then *f* fluid cannot flow to this node.

According to these conditions – if necessary – minimal capacity may also be defined. Accordingly, inter-user entropy can be calculated based on eq. (8).

3.2.2. Transactions

Let

$q_i(k, j, f, f')$ be the measure of a unit of the transformation of *f* fluid to j^{th} node received from k^{th} node. The transformed fluid is f'.

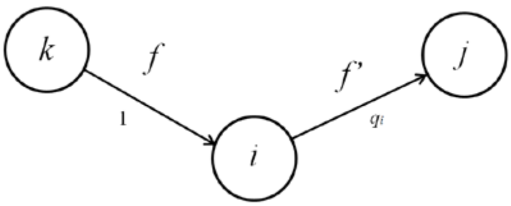

Figure 3. $q_i(k, j, f, f')$ *function*

$t_i(f, f')$ be the time of transmission of *f* fluid to f' fluid (per unit)

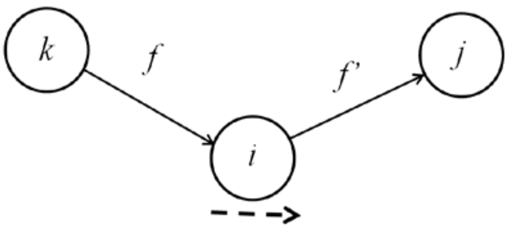

Figure 4. $t_i(f, f')$ *function*

$f_i(k, j, f)$ be the transformation of *f* fluid received from *k*th node to *j*th node (meaning that *f* fluid received from *j*th node transforms into what fluid). Its quantity is determined by $q_i . f' = f_i(k, j, f)$

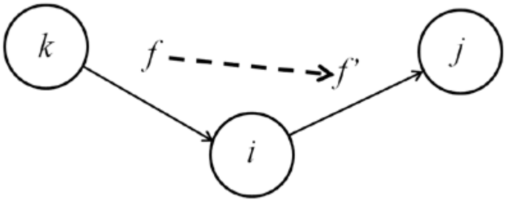

Figure 5. $f_i(k, j, f)$ function

$k_i(f, f')$ be the cost of the transformation of *f* fluid to f'.
$s_i(k, j, f)$ be the subject of the flow of *f* fluid received from *k*th node to *j*th node. This is an empirically determinable function, however it is also applicable.

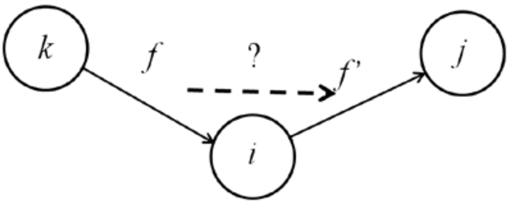

Figure 6. $s_i(k, j, f)$ function

3.2.3. Fluid merging. Fluid merging can be specified with the application of $\mathbf{d_i}$, $\mathbf{t_i}$, $\mathbf{q_i}$ functions of two of the same fluids, thus it does not require any further definitions.

3.2.4. Fluid splitting. Fluid splitting can be specified similarly with the given functions. As for example at $\mathbf{q_i}(\mathbf{k}, \mathbf{j}, \mathbf{d})$ function fluid can be easily split by giving two different *j*.

3.3. Fluid flow model. Let (*i*,*j*) ordered pair denote the direction of the fluid flow, where *i* shall be the sign of the initial node of *f* fluid and *j* be the sign of the target node. In this sense every fluid flow can be represented by a separate ordered pair.

Let us assign to every (*i*,*j*) ordered pair an:
$E_{n_f}(i, j)$ matrix (entropy matrix block)
$E_{q_f}(i, j)$ matrix, which shows the weight of a given *f* fluid flowing from *i*th node to *j*th node
$E_{t_f}(i, j)$ matrix, which shows the period of time of a given *f* fluid flowing from *i*th node to *j*th node
$E_{k_f}(i, j)$ matrix, which shows the cost of a given *f* fluid flowing from *i*th node to *j*th node

It is advisable to introduce an:

$E_f(i,j)$ bivariate matrix, which has a value 1 if *f* fluid flows from *i* to *j*, and 0 if there is no flow.

3.4. The fluid process model. Based on above definitions of fluid, they can even flow in batches. Then

$b(k,j,l,f)$ is the batch of *f* fluids flowing from *i*th node to *j*th node (where *f* vector contains the serial numbers of the fluids that are included in the batch). Batch flows can be also described by the given above and easily generalized functions.

3.5. Unknown data.

Let

x_{ijf} be the weight of *f* fluid flowing from *i*th node and arriving to *j*th node.

y_{ijf} be the weight of *f* fluid flowing from *i*th node and departing to *j*th node.

3.6. Condition system.

1. The changing volume of fluid *f* in relation (*i*,*j*) (except for input nodes)

$$x_{jif} = \mathbf{E}_{q_f}(i,j) \cdot \mathbf{E}_f(i,j) \cdot y_{ijf} \;\; i \notin \{1,\dots,n_p\}, f \in \{1,\dots,m\} \tag{1}$$

2. The total volume of fluid *f* in input nodes is equal to F_i^I:

$$\sum_{j=1}^{n} x_{ijf} = F_i^I \;\; i \in \{1,\dots,n_p\}, f \in \{1,\dots,m\} \tag{2}$$

3. The total volume of fluid *f* in output is nodes less than or equals to F_i^O:

$$\sum_{j=1}^{n} y_{jif} \le F_i^O \;\; i \in \{n_p+1,\dots,n_p+n_k\}, f \in \{1,\dots,m\} \tag{3}$$

4. Mandatory rule of node *i* for fluid *f* is:

$$sgn\left(\sum_{j=1}^{n} x_{jif}\right) \ge h_i(f) \;\; i \in \{1,\dots,n\}, f \in \{1,\dots,m\}. \tag{4}$$

5.

$$y_{ijf_i(k,j,f)} = q_i\big(k,j,f,f_i(k,j,f)\big) \cdot x_{kif} \;\; i \notin \{1,\dots,n_p\}, f \in \{1,\dots,m\}, j \in \{n_p+1,\dots,n_p+n_k\}. \tag{5}$$

6. Not any *f* fluid can be transformed or flown through a given node than the maximum capacity of that node:

$$\sum_{f\ fluid\ entering\ to\ i\in j} x_{ijf} \le p_{max_j}(f), f \in \{1,\dots,m\} \tag{6}$$

7. The sum of the continuation without any formal change of an entering f fluid and other fluids transforming to f must be equal to the sum of leaving f fluids from a node.

$$\sum_{f\ fluid\ entering\ to\ k\in j} x_{kjf} q_j(k,j,f,f) + \sum_{f_i(k,j,f')=f} x_{kjf'} q_j\big(k,j,f,f_i(k,j,f')\big) = = \sum_{f\ fluidum\ flowing\ from\ i\in j} y_{ijf}\ , f \in \{1,\dots,m\}. \tag{7}$$

3.7. The objective function. Our objective in this system is to produce expected fluids on outputs flowing from inputs in required quality and weight. All of this is done by the minimization of the system's internal entropy, lead time and cost. Accordingly, the structure of the objective function is as follows:

$$c(D^I, D^O, P) = \lambda_1 K(D^I, D^O) + \lambda_2 T(D^I, D^O) + \lambda_3 H(D^I, D^O) \to min. \tag{8}$$

where

$\mathrm{K}(\mathrm{D}^\mathrm{I}, \mathrm{D}^\mathrm{O})$ is the cost of input and output fluid flows according to the system,

$$K(F^I, F^O) == \sum_{f=1}^{m} \sum_{j=1}^{n} \sum_{i=n_p+1}^{n} \mathbf{E}_{v_f}(i,j) \cdot x_{jif} + + \sum_{f=1}^{m} \sum_{j=1}^{n} \sum_{i=j+1}^{n} \sum_{k=1}^{n} k_i(f,f') \cdot q_j\big(k,j,f,f_i(k,j,f')\big) \cdot sgn\big(x_{jif}\big). \tag{9}$$

$\mathrm{T}(\mathrm{D}^\mathrm{I}, \mathrm{D}^\mathrm{O})$ is the lead time of input and output fluid flows according to the system,

$$T(F^I, F^O) == \sum_{f=1}^{m} \sum_{j=1}^{n} \sum_{i=n_p+1}^{n} \mathbf{E}_{t_f}(i,j) \cdot x_{jif} + + \sum_{f=1}^{m} \sum_{j=1}^{n} \sum_{i=j+1}^{n} \sum_{k=1}^{n} t_i(f,f') \cdot q_j\big(k,j,f,f_i(k,j,f')\big) \cdot sgn\big(x_{jif}\big). \tag{10}$$

$\mathrm{H}(\mathrm{D}^\mathrm{I}, \mathrm{D}^\mathrm{O})$ is the process entropy of input and output fluid flows.

The fulfilment of output requests is limited by the conditions, so it may occur that there is no possible solution to the problem. The λ_i's of the objective function are normalized scalars allowing us weighting of each component according to their importance during the investigations. If a certain $\lambda_\mathrm{i} = 0$, then the aspect is not included in the examination.

Our investigations basically deal with the third component. The primary $\lambda_1 = \lambda_2 = 0$, and $\lambda_3 = 1$ assumed. Accordingly a P process with $\mathrm{D}^\mathrm{I}, \mathrm{D}^\mathrm{O}$ input and output parameters in accordance with the previous chapter can be considered as follows:

$$H(D^I, D^O) = -\log_2 \min\{P_{Oc}(f) | f \in O(P)\} \tag{11}$$

4. Structure of the system

The system consists of nodes and fluid flows interpreted among them (batch of fluid flows can be considered as fluid processes). It can be described as a controlled graph containing circular paths wherein edges have no values, but rather functions. The objective is the circulation of the fluid from certain inputs to given outputs. This is done while minimizing the given objective function in such a manner that the fluid must pass over compulsory nodes.

These nodes can also be considered as special Mealy-automatons. In contrast to the classical Mealy automaton theory herein stated changes are not necessarily performed one after the other. However, parallel activities may occur. An important condition is that the criterion of discrete time scale is met. Time, though, is allocated to status changes [see $t_i(k, j, d)$ function] that affect the launch of a following process and also the whole lead time.

5. Results and conclusions

To determine and model the operation of a process based system the first and most important task should be the conceptualization of the terms of the related discipline. In our research there have been many undefined and misunderstood terms and comprehension problems have occurred. In order to prepare the modelling phase of our research and to lay the foundations of logistification, this conceptualization is inevitable. Accordingly, the fluid flow based technical and mathematical model of a process system is prepared. A simulation built on this can be carried out to examine and reorganize an arbitrary process system from a flow perspective. A model for nodal process and user entropy is given in this paper. A full mathematical approach is also given on a service process-reengineering concept. This is combined with the development of classical Mealy automaton theory that now fit seven for service processes.

Acknowledgements

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Literature

[1] Eichengreen, B.; Gupta, P. (2013) *The two waves of service-sector growth*. Oxford Economic Papers, Vol. 65(1) pp. 96-123.

[2] Buavaraporn, N. (2010) *Business Process Improvement Methodology Adoption For Improving Service Quality*. Case Studies Of Financial

[3] *Institutions In Thailand*. University of Nottingham. CIA. (2013) The World Factbook 2013-14. Washington, DC.

[4] Sánchez-Rodríguez, C.; Hemsworth, D.; Martínez-Lorente, Á. R.; Clavel, J. G. (2006) *An empirical study on the impact of standardization of materials and purchasing procedures on purchasing and business performance*. Supply Chain Management, Vol. 11(1) pp. 56-64.

[5] Wüllenweber, K.; Weitzel, T. (2007) *An empirical exploration of how process standardization reduces outsourcing risks*. In Proceedings of the 40th HICSS. Waikoloa, Big Island (HI), USA.

[6] Gubán, M. (2005) *Matematical modelling 1/3. (in Hungarian)*. BGF-PSzFK SI, Salgótarján

[7] Hillier, F. S.; Lieberman, G. J. (1994) *Introduction to the operation research* (*in Hungarian*). Budapest: LSI Oktatóközpont

[8] Williams, H. P. (1985) *Model Building in Mathematical Programming*. 2nd ed., New York: Wiley

[9] Lawler, E. L. (1982) *Combinatorial Optimization: Networks and Matroids* (*in Hungarian*). Budapest, Műszaki Könyvkiadó

[10] Kása, R.; Gubán, M.; Gubán, Á.; Hua, N. S.; Réthi, G. (2014) *L.O.S.T.: Logistification of Coffee Services AIB-LAT*. Academy of International Business - Latin American Chapter, Medellín, Colombia

[11] Gubán, Á.; Kása, R.; Gubán, M. (2014) *The Theory of Perception Driven Process Logistification.* Working Services on Production Economics Innsbruck

[12] Gubán, Á.; Kása, R. (2013) *Service Logistics: Logistification of service processes*. Advanced Logistic Systems. Theory and Practice, Vol. 7(1) pp. 43-50. HU ISSN 1789-2198

[13] Gubán, Á.; Kása, R. (2013) *Conceptualization of fluid flows of logistificated processes*. Advanced Logistic Systems Theory and Practice, Vol. 7(2) pp. 27-34. HU ISSN 1789-2198

[14] Hua, N. S.; Gubán, M. (2013) *A data mining method for the solution of fluid-flow problem*. Advanced Logistic Systems. Theory and Practice, Vol. 7(2) pp. 67-76. HU ISSN 1789-2198

[15] Gubán, Á.; Kása, R. (2013) *A Literature Based Review of Business Process Amelioration Methods and Techniques Regarding Service Orientation*. Journal of Advanced Managemet Sciences, Vol. 2(1) pp. 230-235.

[16] Gubán, Á.; Kása, R. (2013) *Service logistics: Logistification of service processes* (*in Hungarian*). A Virtuális Intézet Közép-Európa Kutatására Közleményei, Vol. 6**(**1-2) pp. 479-489.

[17] Bányai, T. (2012) *Direct shipment vs. Cross docking*. Advanced Logistic Systems. Theory and Practice, Vol. 6(1) pp. 83-88. HU ISSN 1789-2198

[18] Bányai, Á. (2012) *Intermodal optimization with harmony search*. Advanced Logistic Systems Theory and Practice, Vol. 6(1) pp. 57-62. HU ISSN 1789-2198