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THE FOUCAULT-PENDULUM

SÁNDOR SZALADNYA[1]

Abstract: In the material handling, there are many machines in which the goods can swing during transport or loading. The greatest impact of this phenomenon is appearing during the operation of different cranes, where the goods are moving as a pendulum. In 1851, Jean Foucault presented a strand-pendulum, which was used to demonstrate the rotation of the Earth. This Foucault-pendulum and its describing equations can help to understand the behaviour of the swinging bodies and the rules which influence their movement. In this paper, the author presents the importance and the essential formulas to describe the behaviour of a pendulum, using the data of the Foucault-pendulum built at the main entrance of the University of Miskolc as a reference object.
Keywords: *swinging, absorption, Coriolis-force, load-swinging*

1. INTRODUCTION

In 1851, Jean Foucault presented a strand-pendulum, which was used to demonstrate the rotation of the Earth. This Foucault-pendulum and its describing equations can help to understand the behaviour of the swinging bodies and the rules which influence their movement. In the material handling, there are many machines in which the goods can swing during transport or loading. The greatest impact of this phenomenon is appearing during the operation of different cranes, where the goods at the end of the lifting cable are moving as a pendulum. A Foucault-pendulum built in 2000 at the main entrance of the University of Miskolc [1], which is a suitable reference object to apply and analyse the pendulum-movement.

2. JEAN FOUCAULT

Jean Foucault was born on 18 September 1819 in Paris, in a rich family. In his 10th year, he started his studies in the “Colleg Stanislas” in Paris, but because of his non suitable diligence, he was taught away form the school. After it, he finished his studies at private teacher. He also abandoned his medical study at the University of Paris due to a blood phobia.

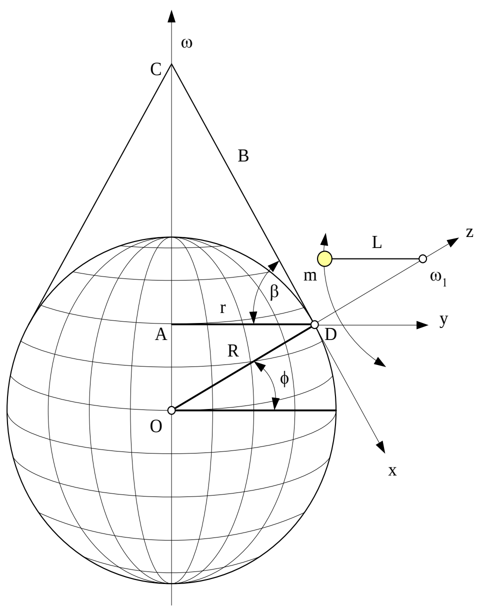
1 Prof. emeritus, University of Miskolc
altemer@uni-miskolc.hu
3515 Miskolc-Egyetemváros, Hungary

Foucault started to work in 1844, in his 25th year, he was scientific reporter at a journal and dealt with the refraction and the determination of the transmission speed of the light. He and Armand Fizeau developed an apparatus with gears and rotating mirror to measure the speed of the light. The gyroscope is also Jean Foucault's invention.

In 1851, Foucault presented a 68 meters long strand-pendulum to the wide audience, which could be used to demonstrate the rotation of the Earth spectacularly. This pendulum as Foucault-pendulum was entered into the History.

3. UNDAMPED PENDULUM MOVEMENT

To describe the movement of a mathematical pendulum, we have to analyse the movement of a weightless and inflexible material point effected by the gravity force. In this analysis we are dealing with only the flat pendulum and aside from the effects of the Earth's rotation [3]. Movement, location and data of the Foucault-pendulum built at the University of Miskolc can be seen in *Figure 1* and *2*.



Data:

- $R = 6.378 \cdot 10^6$ m
- $\phi^M = 48.1^o$
- $r = R \cdot \cos\phi^M = 4259.4$ km
- calendar day:
  $T_n = 24 \cdot 3600 = 86400$ s
- angular speed of the Earth:
  $\omega_F = 7.268 \cdot 10^{-5}$ 1/s

$$\omega_F = \frac{2 \times \pi}{T_n} = \frac{2 \times 3.14}{24 \times 3600} = 7.268 \times 10^{-5} \frac{1}{s}$$

- gravity acceleration:
  $g = 9.832 - 0.052 \cdot \cos\phi^M =$
  $= 9.81$ m/s^2

Figure 1. Geometric data of the Foucault-pendulum [3]

Basic equations to describe the movement of the pendulum:

- moment of the inertia force calculated to the axis of rotation:

$$M_t = m \cdot L^2 \cdot \frac{d^2}{dt^2} \alpha \tag{1}$$

- moment of the weight force calculated to the axis of rotation:

$$M_g = -L \cdot m \cdot g \cdot \sin\alpha \tag{2}$$

For small angles $sin\alpha = \alpha$, so

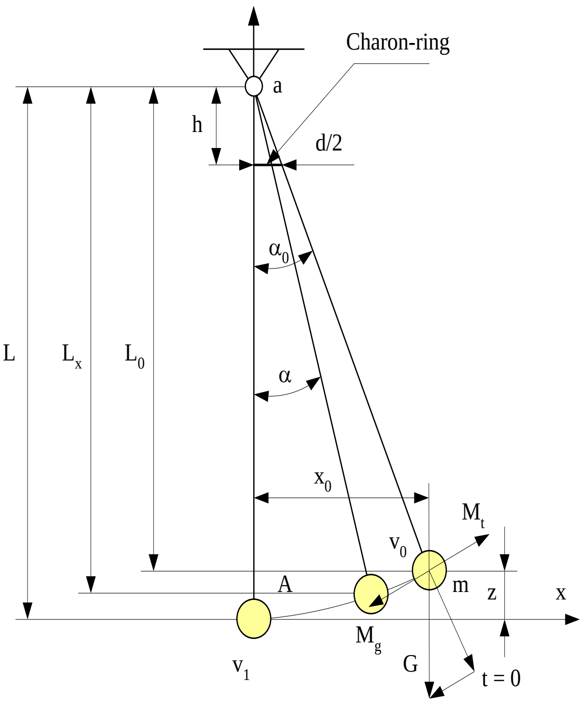
$$M_t = M_g \tag{3}$$

$$\frac{d^2\alpha}{dt^2} + \frac{g}{L}\cdot\alpha \cong 0 \tag{4}$$

$$\frac{g}{L} = \omega^2, \omega = \omega_0 \tag{5}$$

$$\frac{d^2\alpha}{dt^2} = -\omega^2 \cdot \alpha \tag{6}$$

Equation (6) is the differential equation of the undamped harmonic oscillation.



Data of the pendulum at Miskolc:

- α = 3–7º
- length of the strand: L = 10.4 m
- swinging mass: m = 42 kg

Figure 2. Forces effect the pendulum [2]

Dumped harmonic oscillation can be done by a body, if the size of the acceleration is proportional to the movement, but its direction is opposite. This kind of oscillation is valid for the mathematical pendulum. In our analysis we use Cartesian coordinate system.

As it can be seen in *Figure 2* and *3* (Figures are not scaled) L = 10.4m, N = h = 0.16m, d = 0.016 m – diameter of the Charon-ring (see *Figure 2*), which localizes the motion of the pendulum. If the amplitude of the swinging is small, when x_0/L = 0.052 and Z/L = 0.0014 values are small, the end point of the pendulum is approximately follow angle x (see *Figure 3*).

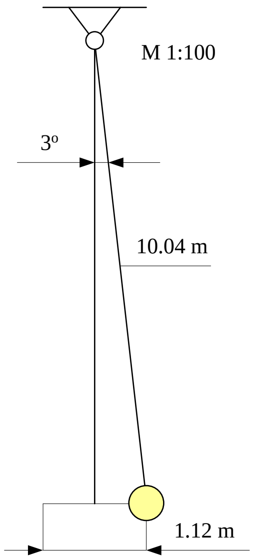


Figure 3. Effect of the Charon-ring

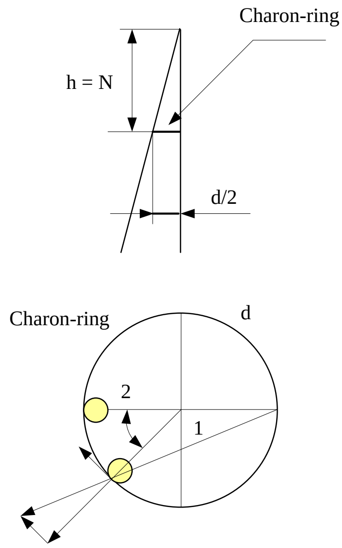


Figure 4. Geometry of the Charon-ring

Using the described values and $\alpha_0 = 3^o$:

$$tg\alpha = \frac{d/2}{N} = 0.05 \quad I_v = L \cdot \frac{3 \cdot \pi}{180} = 0.5445m$$

$$x_0 = L \cdot \sin\alpha_0 = 0.544m \quad L_0 = L \cdot \cos\alpha = 10.385m$$

$$Z = L - L_0 = 0.015m \quad v_1 = \sqrt{2 \cdot g \cdot Z} = 0.542m/s$$

Movement function of the pendulum in direction x – horizontal movement above the table- in time can be described by the next equation with acceptable approximation:

$$\ddot{x} = -\omega^2 \cdot x \tag{7}$$

Equation (7) is a linear differential equation of the second order with constant coefficients, which is suitable to analyse the undamped pendulum movement. Its characteristic equation is $\lambda^2 + \omega^2 = 0$, its roots are $\lambda_{1,2} = \pm\ \omega i$. General solution of the differential equation is

$$x(t) = c_1 \cdot \cos\omega \cdot t + c_2 \cdot \sin\omega \cdot t \tag{8}$$

If we apply equation (8) for the pendulum taking *Figure 2* into consideration and determine the values of c_1, c_2 constants:

$$\frac{d}{dt}x(t) = v(t) = -c_1 \cdot \sin(\omega \cdot t) \cdot \omega + c_2 \cdot \cos(\omega \cdot t) \cdot \omega \tag{9}$$

If t = 0, x(0) = 0.544, $v_0 = 0$, $c_2 = 0$, then $\omega = \sqrt{\frac{g}{L}} = 0.971$.

Taking the above mentioned into account:

$$x(t) = c_1 \cdot \cos(\omega \cdot t) \quad (10)$$

$$v(t) = c_1 \cdot \omega \cdot \sin(\omega \cdot t) \quad (11)$$

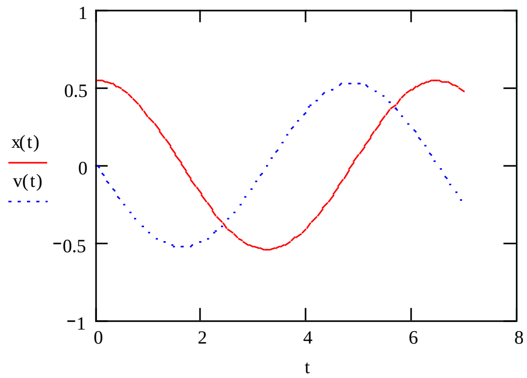


Figure 5. Movement and velocity of the pendulum in time

4. Damped Pendulum movement

If the energy of the movement of the pendulum is absorbed by anything (e. g. friction), then we talk about damped pendulum movement. In this case the pendulum movement is proportional to the acceleration and the velocity, but its direction is opposite to both of them.

4.1. Equations for damped pendulum movement

Differential equation to describe the movement:

$$m \cdot \frac{d^2x}{dt^2} = -\omega^2 \cdot m \cdot x - 2 \cdot s \cdot \frac{dx}{dt} \quad (12)$$

where

- *m* – mass of the moving body [kg],
- *s* – dumping coefficient.

If we divide equation (12) by *m* and use the *s/m* = *k* marking, as a resistance factor, then

$$\frac{d^2x}{dt^2} + 2 \cdot k \cdot \frac{dx}{dt} + \omega^2 \cdot x = 0 \quad (13)$$

where $\omega \geq 0$ is a constant and *k* is a positive constant value.

The characteristic equation of the homogeneous differential equation of the second order with constant coefficients is

$$\lambda^2 + 2 \cdot k \cdot \lambda + \omega^2 = 0 \tag{14}$$

and its roots are

$$\lambda_{1,2} = -k \pm \sqrt{k^2 - \omega^2} \tag{15}$$

If $k < \omega$, i. e. $\sqrt{k^2 - \omega^2} < 0$, then the roots of the characteristic equation are complex values, so

$$\lambda_{1,2} = -k \pm i \cdot \sqrt{k^2 - \omega^2} < 0 \tag{16}$$

This case will be used for the following analysis of the pendulum.

In case of dumped movement, two other equations are required to determine the values of c_1, c_2 constants in the general solution:

$$x(t) = e^{-k \cdot t} \cdot \left[c_1 \cdot \cos\left(\sqrt{\omega^2 - k^2} \cdot t\right) + c_2 \cdot \sin\left(\sqrt{\omega^2 - k^2} \cdot t\right) \right] \tag{17}$$

$$\begin{aligned} \frac{d}{dt} x(t) &= -k \cdot e^{-k \cdot t} \cdot \left[c_1 \cdot \cos\left(\sqrt{\omega^2 - k^2} \cdot t\right) + c_2 \cdot \sin\left(\sqrt{\omega^2 - k^2} \cdot t\right) \right] + \\ &+ e^{-k \cdot t} \cdot \left[-c_1 \cdot \sin\left(\sqrt{\omega^2 - k^2} \cdot t\right) \cdot \sqrt{\omega^2 - k^2} + c_2 \cdot \cos\left(\sqrt{\omega^2 - k^2} \cdot t\right) \cdot \sqrt{\omega^2 - k^2} \right] \end{aligned} \tag{18}$$

It can be read from equation (17) and (18), that the pendulum movement is a periodic function in time, its amplitude is decreasing exponentially with the increasing of the *t* value, which results the damping effect. If we release the pendulum in state "0", its movement will be as it can be seen in *Figure 6* and *7*. In case of small amplitude, x(290·T) = 0,1 mm, the kinetic energy of the pendulum is so low, that the movement of the surrounding air can also affect the pendulum movement (v = 0.048 m/s). The value of the "k" factor used in the equations is calculated on previous observations, which will be corrected based on practical measures.

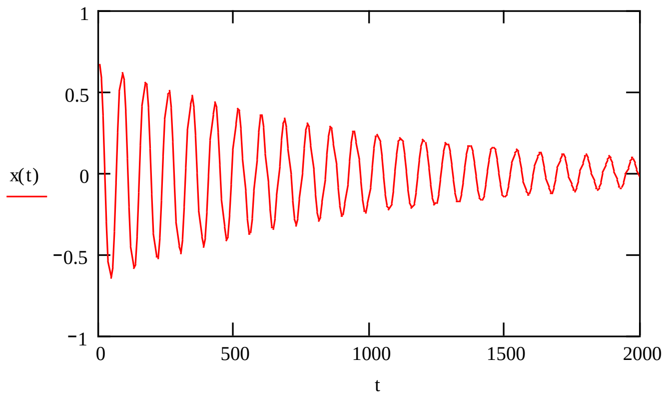


Figure 6. Pendulum movement

In the knowledge of the values of the constants, the damped pendulum movement and velocity can be depicted in time *(Figure 6* and *7).*

The values of the constants are k = 0.001, ω = 0.971 1/s, c_1 = 0.669, $c_2 = \frac{-k \cdot c_1}{\sqrt{\omega^2 - k^2}}$.

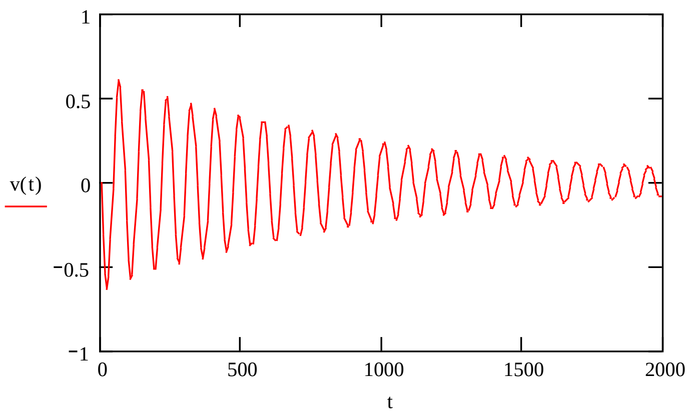


Figure 7. Velocity of the pendulum

As the results of the above described process are not sufficient to the further analysis of the pendulum movement, so we have to introduce new equations.

4.2. Determination of the period of the pendulum movement

The period of the movement of the Foucault-pendulum built in Miskolc, based on the equations in the related literatures [5]:

$$T = 4 \cdot \sqrt{\frac{L}{g}} \cdot \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - x^2 \cdot \sin(\alpha)^2}} d\alpha \tag{19}$$

Applying the data of the pendulum (α = 3°-4°, L = 10.4 m, g = 9.81 m/s^2, ω = 0.971 1/s) and x = sin(α_0/2) the calculated period of the pendulum movement is T = 6.47 s.

Determination of the period is even simpler, if α < 8°:

$$T = 2 \cdot \pi \cdot \sqrt{\frac{L}{g}} \tag{20}$$

In this case, the calculated period of the movement is T = 6.469 s.

We can establish that the period of the pendulum movement is not depending on the mass and the amplitude of the pendulum. It means that the same pendulums in length with different masses and amplitudes (small) on the same location of the Earth have the same movement period. This is the rule of isochronous time which was discovered by Galilei, which will be taken into account in the analysis of the damped pendulum movement, for the evaluation of the measuring results. Confirm this rule using equation (17) with suitable transformations and variable amplitudes:

$$x(t,c_1) = e^{-k\cdot t}\cdot\left[c_1\cdot\cos\left(\sqrt{\omega^2-k^2}\cdot t\right)+\frac{-k\cdot c_1}{\sqrt{\omega^2-k^2}}\cdot\sin\left(\sqrt{\omega^2-k^2}\cdot t\right)\right] \tag{21}$$

We can see in Figure 8, that the movement period is the same for all amplitude variations.

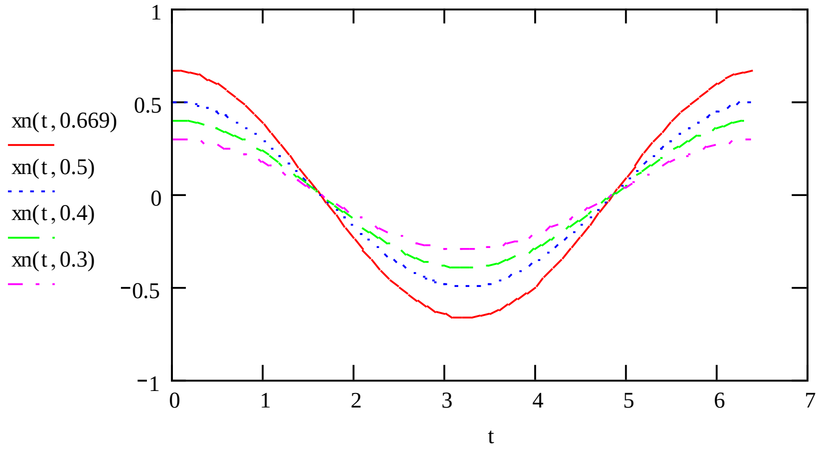


Figure 8. Effect of the amplitude changing to the period of the movement
k = 0.001, ω = 0.971 1/s, T = 6.47 s

4.3. Comparison of the calculated and measured values of the period of the pendulum movement, determination of the resistant factor

Curves resulted by the equations described before show that after releasing, the kinetic energy of the pendulum is decreasing. To keep the pendulum in continuous moving, it is required to refill the energy, so on certain places, magnetic push have to be used.

As our objective is to determine the value of the "k" resistant factor, so we realized our measuring after switching the magnet off. During the motion we measured the decreasing values of the amplitude of the pendulum. Starting value of the amplitude $x_0 = 0.669$ m was at t = 0 s.

Applying the results of the measures *(Table I)* and equations (17) and (18) we determined the real value of the *t* factor. Solving equation (17) with the new values we got *Figure 9*, where k = 0.002, ω = 0.971 1/s, $c_1 = 0.669$.

As it was described before, the decreasing of the amplitude does not influence the period of the movement *(Figure 8)*, so calculating the movement on integer multiples of the period x(n·T) results an envelope for the decreasing of the amplitude *(Figure 10)*.

Calculated values of the movement are based on equation (17) and involved in *Table II*. Curve x(n, T) is the upper envelope of the x(t) function. This curve will be compared to the envelope based on the measured (x_1) values *(Figure 11)*.

Last data of the measuring was $t_1 = 3600$ s (556.4·T), because at this point the kinetic energy of the pendulum was so low that the movement of the surrounding air effected it and the pendulum movement was transformed into spherical pendulum movement.

Table I.
Measuring results

Measure	x_1 [m]	t_1 [s]
1.	0.669	0
2.	0.64	120
3.	0.625	300
4.	0.61	360
5.	0.6	480
6.	0.588	540
7.	0.575	660
8.	0.563	780
9.	0.544	900
10.	0.525	1020
11.	0.506	1200
12.	0.485	1440
13.	0.461	1680
14.	0.438	1920
15.	0.414	2160
16.	0.398	2400
17.	0.376	2640
18.	0.357	2940
19.	0.341	3180
20.		3600

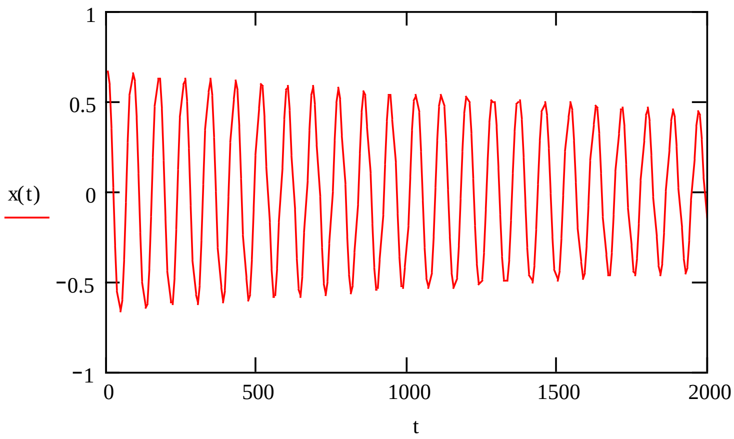


Figure 9. Decreasing of the amplitude of the movement with the corrected k value
$x(0)$ =0.669 m, $x(T)$ = 0.668 m, $x(2\cdot T)$ = 0.667 m, $x(500\cdot T)$ = 0.322 m

Results based on equation (17) have suitable similarity to the measured values (Figure 11), so the value of the resistant factor can be recalculated

$$k = \frac{s}{m}, \quad k = 0.0002$$

Table II.
Calculated values of the upper envelope

	x [m]	**t [s]**
1.	0.669	0
2.	0.627	323.5
3.	0.586	647
4.	0.547	970.5
5.	0.510	1294
6.	0.474	1618
7.	0.440	1941
8.	0.408	2265
9.	0.378	2588
10.	0.349	2912
11.	0.322	3235
12.	0.296	3559
13.	0.272	3882
14.	0.249	4206
15.	0.228	4529

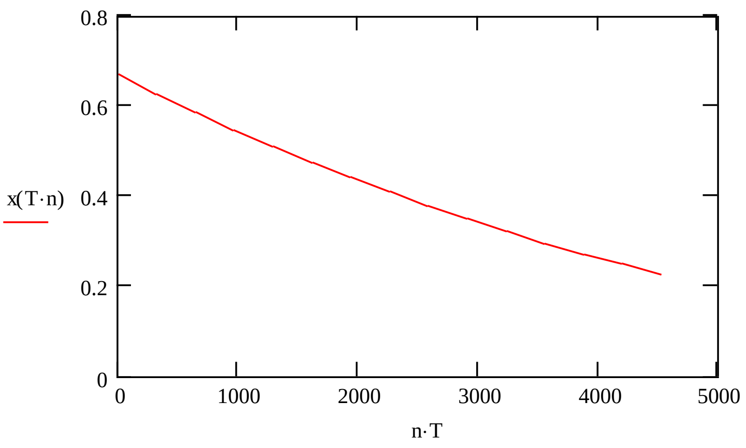


Figure 10. Upper envelope of the calculated amplitude values

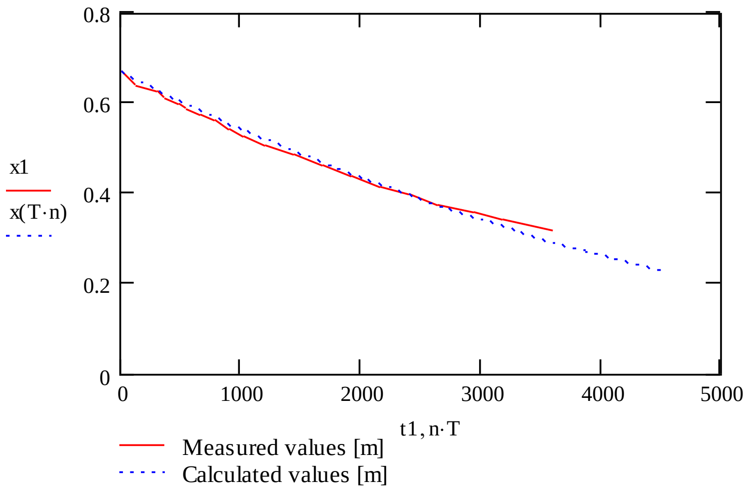


Figure 11. Comparison of the measured and calculated values

5. PENDULUM MOVEMENT ABOVE THE TABLE

5.1. Influencing forces

Figure 12 and *13* show the table and the above swinging pendulum located at the north side of the Earth [3]. The moving body (pendulum), related to the table, is influenced by not only the radial force but also another force which is an inertial force called Coriolis–force. In a rotating reference frame (see *Figure 14*), a force, perpendicular to the velocity, effects to the pendulum starting from point (1), which deflects it into point (2). This is the Coriolis-force.

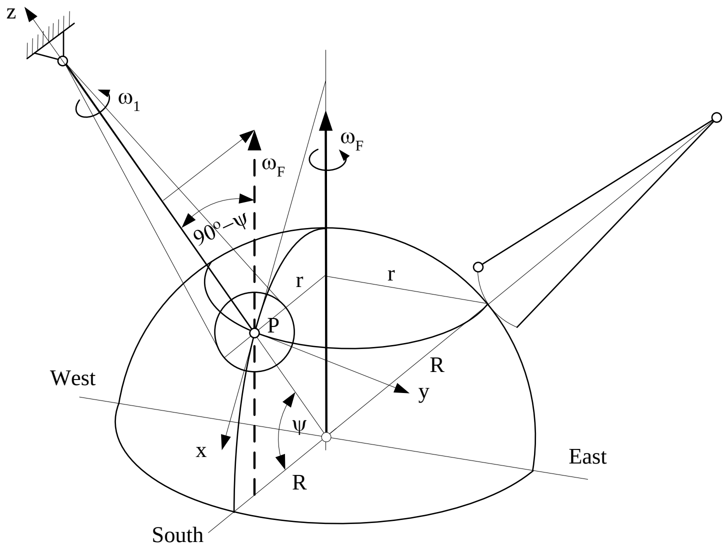


Figure 12. Forces effect the pendulum

In return phase, the same force influences the movement from point (2) to (3). The observer see that the plane of the pendulum movement does not change, but the table is moving. The movement of the table is proportional to the rotation of the Earth.

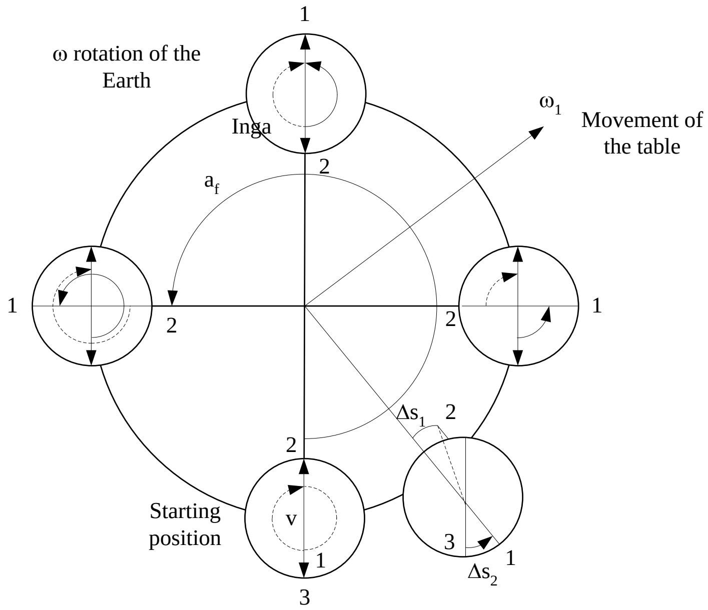


Figure 13. Phases of the pendulum movement [3]

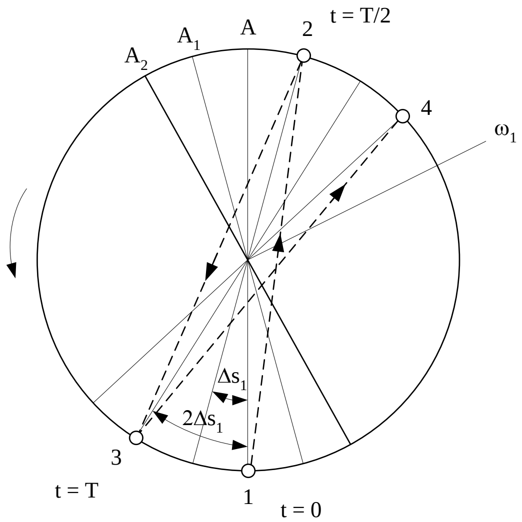


Figure 14. Effect of the Coriolis-force

In *Figure 15*, the changing of the location of the table and the pendulum through the rotation of the Earth can be followed. At starting of the pendulum movement (1) the time is zero (t = 0).

The time of the pendulum movement between point (1) and (2) is T/2. The value of the table movement can be calculated using *Figures 12–15* with the following data:

D – diameter of the circle followed by the end point of the pendulum [m]: D = 2·A,
A – amplitude of the pendulum movement [m],
ω – angular speed of the Earth [1/s]: $\omega = 7.29 \cdot 10^{-5}$,
ω_1 – angular speed of the table [1/s],
T – period of the pendulum movement [s]: T = 6.47 s,
$\phi = 48.1^o$.

$$\frac{\omega_1}{\omega} = \cos(90 - \phi)$$

$$\omega_1 = \omega \cdot \sin(\phi) = 5.43 \cdot 10^{-5}$$

Based on *Figure 13*:

$$\Delta s_1 = A \cdot \omega_1 \cdot \frac{T}{2}$$

The movement of the table during the period (T) is $2 \cdot \Delta s_1$.

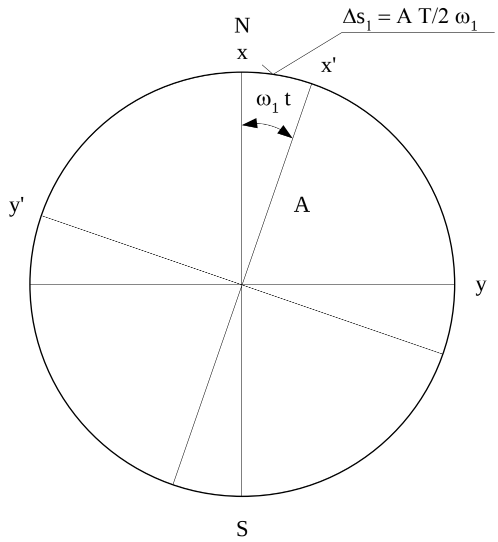


Figure 15. Location of the table and the pendulum during the rotation of the Earth

5.2. Coriolis-force

In a rotating reference frame, the pendulum movement diverges from the table in right direction at the Northern Hemisphere, caused by the rotation of the Earth *(Figures 13–15)*. The perpendicular movement is changing steadily from zero starting velocity. During short Δt times, to a close approximation, the Coriolis-force has to be taken as a constant value into consideration (see [5]). Acceleration of the Coriolis-force is

$$a_c = \frac{2 \cdot \Delta s}{\Delta t^2} \tag{22}$$

where

$$\Delta s = A \cdot \omega_1 \cdot \Delta t \quad (23)$$

$$\Delta t = \frac{A}{v} \quad (24)$$

Taking the above mentioned into account

$$F_c = 2 \cdot m \cdot v \cdot \omega_1 \quad (25)$$

where

F_c – Coriolis-force [N],
Δs – movement [m],
Δt – time [s]: Δt = T/n,
A – amplitude [m],
v – velocity of the pendulum [m/s],
m – mass of the pendulum [kg].

5.3. Effect of the Coriolis-force to the pendulum movement

The angular speed vector ω can be decomposed into a horizontal ω_2 and a vertical vector ω_1. The absolute values of them are

$$\omega_1 = \omega \cdot \sin\psi \quad (26)$$

$$\omega_2 = \omega \cdot \cos\psi \quad (27)$$

Suited to this, the Coriolis-force can also be decomposed into two components:

$$F_c = 2 \cdot m \cdot v \cdot \omega_1 + 2 \cdot m \cdot v \cdot \omega_2 = F_{c1} + F_{c2} \quad (28)$$

The first component means the force – arises on the pendulum moving in horizontal plane – tending to right direction which is perpendicular to the velocity of the Earth at the Northern Hemisphere. The second component is a vertical force in down or up direction, its effect is the changing of the weight of the moving mass.

The pendulum moving in horizontal direction is tending to right direction from its path because of the rotation of the Earth and the Coriolis-force. The amount of the difference can be calculated taking the values in *Figure 16* and *18* into consideration (L = 10.4 m, g = 9.81 m/s^2, m = 42 kg, G = m·g, L_0 = 10.38 m, π = 3.14, A = 0.669 m, $\omega_1 = 5.4275 \cdot 10^{-5}$ 1/s).

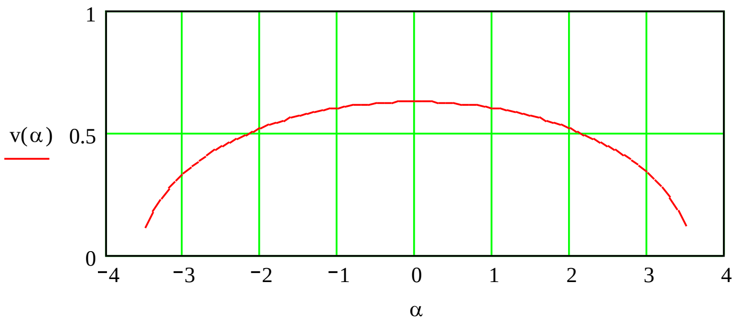


Figure 16. Velocity of the pendulum

Based on Figure 2, the energy equation of the pendulum movement can be described, from which the velocity of the pendulum *(Figure 16)*:

$$v(\alpha) = \sqrt{\left[10.4\cdot\cos\left(\alpha\cdot\frac{\pi}{180}\right)-10.38\right]\cdot 2\cdot 9.81} \tag{29}$$

From *Figure 17* the torque equations to point “0”:

$$M_1 = \left(2\cdot m\cdot v\cdot \omega_1 - m\cdot g\cdot\sin\alpha\right)\cdot L \tag{30}$$

$$M_2 = m\cdot g\cdot k \tag{31}$$

$$\sin\alpha = \frac{k}{L} \tag{32}$$

$$k(\alpha) = L\cdot\sin\alpha \tag{33}$$

Arranging the equations above and applying that

$$M_1 = M_2 \tag{34}$$

$$k(\alpha) = v(\alpha)\cdot\omega_1\cdot\frac{L}{g} \tag{35}$$

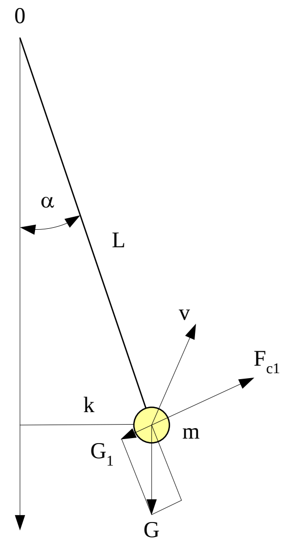


Figure 17. Forces on the pendulum

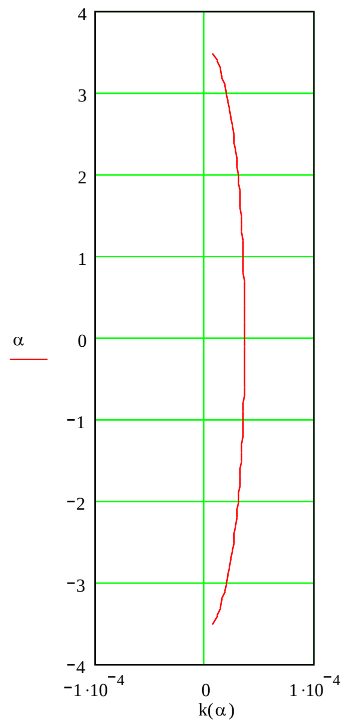


Figure 18. Deviation of the pendulum

Applying equation (29), the diagram presented in *Figure 18* is resulted. The distance from the origin is $k(0)=3.64\cdot 10^{-5}\,m$. We can see in Figure 18 that during the continuous motion, the pendulum does not go through the origin, but the deviation is so small that the observer cannot sense it and see a straight moving.

6. ANOTHER APPROACH OF THE MOVEMENT OF THE FOUCAULT-PENDULUM ABOVE THE TABLE

In this approach, we add the Coriolis-force to the equations of the pendulum movement in the reference frame, and we analyse only small-amplitude-swinging of the pendulum, when x/L and y/L are so small values, that their higher powers are neglected, only the first powers are taken into account. In this approach, the endpoint of the pendulum is moving in a horizontal plane (see [3]). Based on *Figure 1* and *12*:

$$\psi = 48.1^{o} \qquad \omega = 7.29\cdot 10^{-5}\,\frac{1}{s} \qquad \omega_1 = \omega\cdot\sin\psi = 5.4275\cdot 10^{-5}\,\frac{1}{s}$$

The equations of the motion:

$$\frac{d^2x}{dt^2} = 2\cdot\omega\cdot\sin\psi\cdot\frac{dy}{dt}+\lambda\cdot x \tag{36}$$

$$\frac{d^2y}{dt^2} = -2\cdot\omega\cdot\sin\psi\cdot\frac{dx}{dt}-2\cdot\omega\cdot\cos\psi\cdot\frac{dz}{dt}+\lambda\cdot y \tag{37}$$

$$\frac{d^2z}{dt^2} = -g+2\cdot\omega\cdot\cos\psi\cdot\frac{dy}{dt}+\lambda\cdot z \tag{38}$$

$$z=-L \qquad \frac{dz}{dt}=0 \qquad \frac{d^2z}{dt^2}=0 \qquad \lambda=\frac{-g}{L}$$

In equation (38), the component involving ω is very small related to *g*, so it is negligible

$$0=-g-\lambda\cdot L \to \lambda = -\frac{g}{L}$$

Using this simplification, the equations of the motion are

$$\left.\begin{aligned}\frac{d^2x}{dt^2}-2\cdot\omega_1\cdot\frac{dy}{dt}+\frac{g}{L}\cdot x=0\\ \frac{d^2y}{dt^2}+2\cdot\omega_1\cdot\frac{dx}{dt}+\frac{g}{L}\cdot y=0\end{aligned}\right| \tag{39}$$

The equations above can be coupled in one complex equation:

$$u = x+i\cdot y \tag{40}$$

Suited to (40), the equation of the motion is

$$\frac{d^2u}{dt^2}+2\cdot i\cdot\omega_1\cdot\frac{du}{dt}+\frac{g}{L}\cdot u=0 \tag{41}$$

Taking *Figure 15* into account

$$u' = x'+i\cdot u' = u\cdot e^{i\cdot\omega_1\cdot t} \tag{42}$$

After simplification and taking the starting conditions into consideration

$$x = x' \cdot \cos(\omega_1 \cdot t) + y' \cdot \sin(\omega_1 \cdot t) \quad (43)$$

$$y = -x' \cdot \sin(\omega_1 \cdot t) + y' \cdot \cos(\omega_1 \cdot t) \quad (44)$$

$$x' = x_1(t) \cdot \cos(\omega_1 \cdot t) - y_1(t) \cdot \sin(\omega_1 \cdot t) \quad (45)$$

$$y' = x_1(t) \cdot \sin(\omega_1 \cdot t) + y_1(t) \cdot \cos(\omega_1 \cdot t) \quad (46)$$

These equations mean that the axes x' and y' are rotating in a horizontal plane with ω_1 angular speed, related to the observing *x-y* system. Transformations and simplifications can be seen in reference [1].

In (x', y') system, if t = 0, then

$$x' = A \quad y' = 0 \quad \frac{dx'}{dt} = 0 \quad \frac{dy'}{dt} = A \cdot \omega_1 \quad a = A = 0.669m$$

$$x' = a \cdot \cos\left(\sqrt{\frac{g}{L}} \cdot t\right) \quad y' = b \cdot \sin\left(\sqrt{\frac{g}{L}} \cdot t\right)$$

$$\sqrt{\frac{g}{L}} = \omega_0, \quad \frac{b}{a} = \omega_1 \cdot \sqrt{\frac{L}{g}} \Rightarrow b = a \cdot \frac{\omega_1}{\omega_0} = 0.6 \cdot \frac{5.4275 \cdot 10^{-5}}{0.971} = 3.739 \cdot 10^{-5}$$

Applying the above described values and substitute them into the (43) and (44) equations, the (47) and (48) equations are obtained, which describes the pendulum movement as a function of the swinging period (see *Figure 19* and *20*).

$$x(t) = A \cdot \left[\cos(\omega_1 \cdot t) \cdot \cos(\omega_0 \cdot t) + \frac{\omega_1}{\omega_0} \cdot \sin(\omega_1 \cdot t) \cdot \sin(\omega_0 \cdot t)\right] \quad (47)$$

$$y(t) = A \cdot \left[\frac{\omega_1}{\omega_0} \cdot \cos(\omega_1 \cdot t) \cdot \sin(\omega_0 \cdot t) - \sin(\omega_1 \cdot t) \cdot \cos(\omega_0 \cdot t)\right] \quad (48)$$

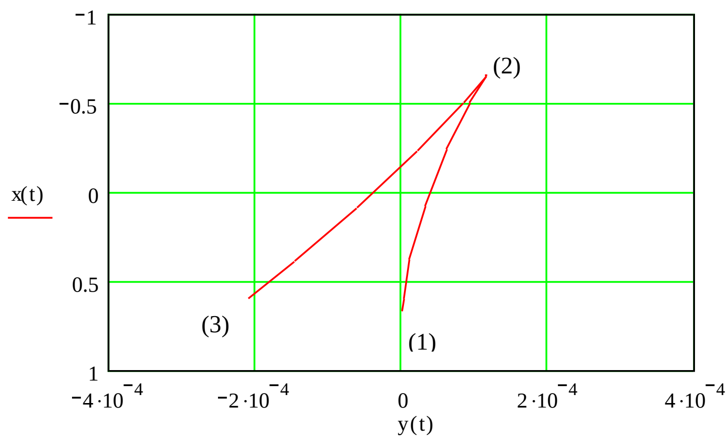


Figure 19. One single period of the pendulum movement
(1) – start of the pendulum, y(0), (2) – y(T/2) = 1.054 · 10^{-4}, (3) – y(T) = –2.107 · 10^{-4}

Figure 19 and *20* present the movement of the pendulum in (x', y') plane, where L = 10.4 m, g = 9.81 m/s^2, A = 0.669 m, ω_1 = 5.4275·10^{-5} 1/s, ω_0 = 0.971 1/s, ω = 7.29·10^{-5} 1/s, T = 6.47 s.

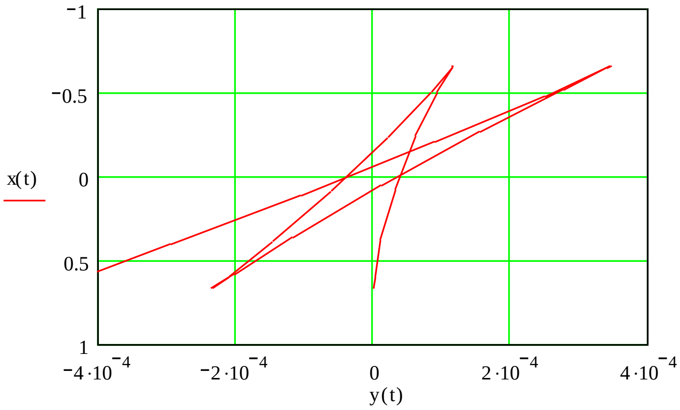


Figure 20. Two periods of the pendulum movement (y(T/4) = 3.353 · 10^{-5})

Transforming equations (45) and (46), *Figure 21* is obtained, which shows that the pendulum is moving along an elliptic line (diagram is not scaled).

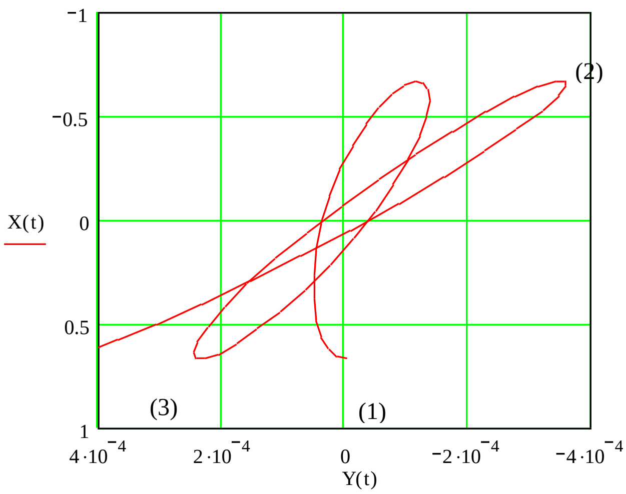


Figure 21. Elliptic characterisation of the pendulum movement
(1) – start of the pendulum, Y(0), (2) – Y(T/2), (3) – Y(T)

In *Figures 19–21*, the pendulum is moving in (x', y') plane. The pendulum is horizontally rotating with ω_1 angular speed, where a = 0.669 m, b = 3.74·10^{-5}, ω_1 = 5.4275·10^{-5} 1/s, ω_0 = 0.971 1/s, ω = 7.29·10^{-5} 1/s, T = 6.47 s.

$$x_1(t) = a \cdot \cos(\omega_0 \cdot t) \tag{49}$$

$$y_1(t) = b \cdot \sin(\omega_0 \cdot t) \tag{50}$$

$$X(t) = x_1(t) \cdot \cos(\omega_1 \cdot t) - y_1(t) \cdot \sin(\omega_1 \cdot t) \tag{51}$$

$$Y(t) = x_1(t) \cdot \sin(\omega_1 \cdot t) + y_1(t) \cdot \cos(\omega_1 \cdot t) \tag{52}$$

Based on the figures the next thesis can be described: *at a given geographical attitude „ψ" of the Earth, the pendulum movement – in case of small amplitudes – follows an elliptic line, and the axes of the ellipse are rotating in a horizontal plane with ω_1 angular speed in north-east direction at the Northern Hemisphere (see* Figures 19–21*).*

Figure 21 is not scaled, but it is suitable to demonstrate the elliptic movement of the pendulum. Based on the equations of Müller [3], and the previously defined values (L = 10.4 m, g = 9.81 m/s^2, a = 0.669 m, $\omega_1 = 5.4275 \cdot 10^{-5}$ 1/s, $\omega_0 = 0.971$ 1/s, T = 6.47 s)

$$b = a \cdot \omega_1 \cdot \sqrt{\frac{L}{g}} = 3.739 \cdot 10^{-5} \qquad \frac{a}{b} = 1.995 \cdot 10^4$$

Figure 22 shows the movement of the pendulum during the time "T". The movement has elliptic character, but the smaller axis of the ellipse is so small that the observer cannot sense it and see a straight moving.

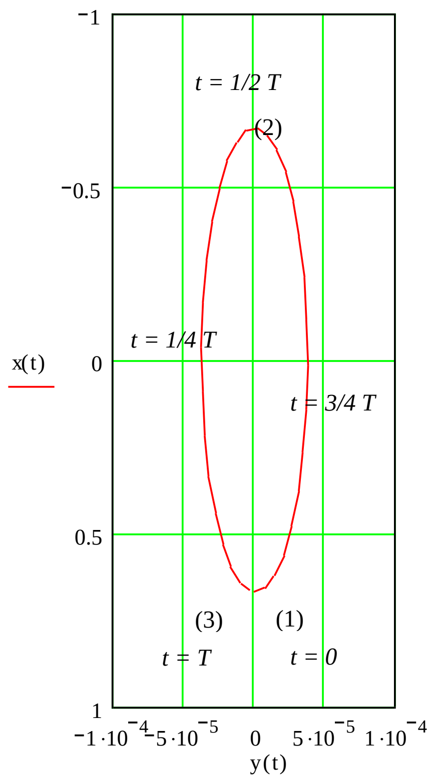


Figure 22. Pendulum movement during the time "T"

7. SUMMARY

Jean Foucault's strand-pendulum, which was used to demonstrate the rotation of the Earth, can help to understand the behaviour of swinging bodies and the rules which influence their movement. As the material handling uses many machines, in which the units can swing during transport or loading, the analysis of the pendulum movement can result many important data for the design of the handling machines and processes. The theoretical method described by the author was confirmed by the data and measuring values of the Foucault-pendulum built in the main reception hall of the University of Miskolc. A possible sequel can be the analysis of bridge cranes, where the pendulum movement directly influences the operation parameters and causes operation problems. It is especially true for automated bridge cranes.

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