MATHEMATICAL DESCRIPTION OF THEORETICAL METHODS OF RESERVE ECONOMY OF CONSIGNMENT STORES

Péter Telek, József Cselényi, György Demeter

University of Miskolc, Department of Materials Handling and Logistics

Abstract: Optimization of the operation of a consignment store in generally is not an easy task. The main problem is the stochastic characterization of the consumption of the products which makes the using of deterministic methods impossible. In our research project we developed a theoretical method which uses historic data of a previous term to determine the optimal values for the parameters of the consignment storing process. Main advantage of the researched method is to give possibility for the designers and operators of the consignment storing process to find (or at least approach) the optimal parameter values which is very difficult by any other way. The new method helps to determine the parameters of the storing process previously as there can be seen in an example.

Keywords: Logistics, stocking, supplying, consignment store

Consignment stores have special situation because of the operation specialties which require setting the stock level near the optimal value. This optimal value is enforced by the operator company (which would like to increase the stored quantity to avoid the lack of products) and also by the manufacturer company (which would like to decrease the stored quantity to reduce the amount of money blocked by the stock).

1. Consignment stores

The calculation, actualization and sustaining of the optimal stock value are in generally not easy tasks and require exact description of the main characteristics of consignment stores and their relations.

The most important characteristics of consignment stores (Figure 1.):

- uploading level: $\underline{R_T}$,
- minimal stock limit: $\underline{R_J}$,
- exhaustion of the stock: ΔQ_F ,
- time of the exhaustion of the stock: T_0 ,
- angle of the exhaustion function: α ,
  which can be calculated in case of linear exhaustion:

$$tg\alpha = \frac{T_0}{\Delta Q_F}, \tag{1}$$

- difference in exhaustion of the angle: $\pm\Delta\alpha$,
    - by the highest exhaustion (if the exhaustion time is the shortest):

$$\alpha_{min} = \alpha - \Delta\alpha \ , \tag{2}$$

    - by the lowest exhaustion (if the exhaustion time is the longest):

$$\alpha_{max} = \alpha + \Delta\alpha \ , \tag{3}$$

- term of the product order: t_R .

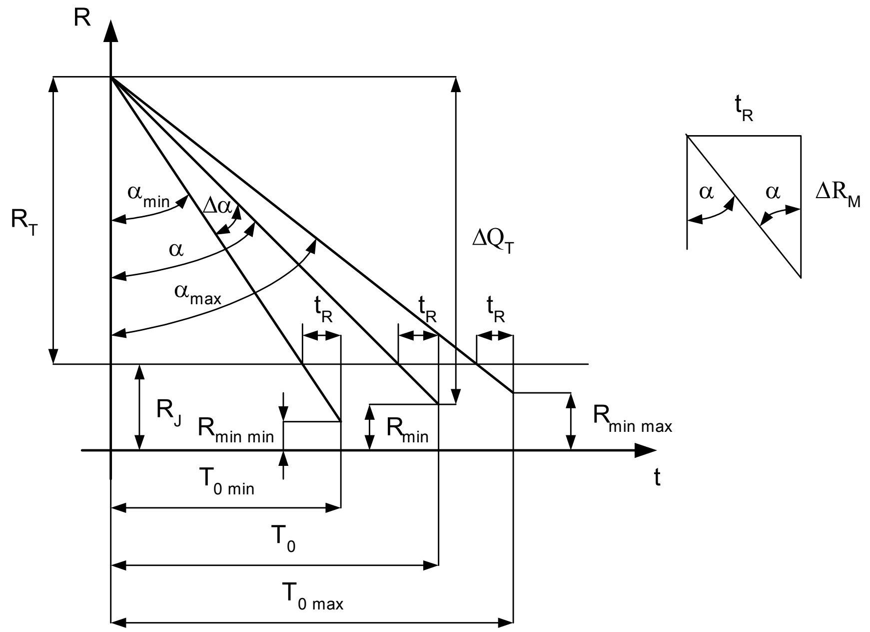

Figure 1. Changing of the angle of the exhaustion at stocks of consignment stores

where

$R_{min\ min}$ - minimal value of the minimal stock levels,
$R_{min\ max}$ - maximal value of the minimal stock levels,
$T_{0\ min}$ - minimal value of the time of the exhaustion,
$T_{0\ max}$ - maximal value of the time of the exhaustion.

2. Analyzing of the stock levels of consignment stores

At the analysis of stock levels of consignment stores if deterministic or prognostic data can not be available, the only one solution is the mathematical-statistical processing of historic data of a previous production term.
In this case, at first we have to analyze periodically the values of

- the angle of the exhaustion function,
- the difference between changings of the angle.

Then based on the analysis of the angle of the exhaustion function the next parameters can be determined:

- uploading level,
- minimal stock limit,
- term of the order.

2.1. Uploading quantities

If we use constant uploading quantities to fill the stock of the consignment store (Figure 1.), the required uploading quantity can be calculated by:

$$\Delta Q_T = R_T - \overline{R}_{\min} = R_T - R_J + \overline{\Delta R_M} = R_T - R_J + \frac{t_R}{tg\alpha}, \tag{4}$$

where

$\overline{R}_{\min}$ - average of the minimal stock values,

$\overline{\Delta R_M}$ - average of the exhaustions below the minimal stock limit.

As the different values of $\Delta\alpha$ are not constant so the uploaded level (R_T) can not be constant, its value will change between the two limit values (minimal and maximal) depend on the minimal stock values.
Limit values of the uploaded level (R_T) are:

- minimal:

$$R_{T\min} = R_{\min\min} + \Delta Q_T < R_T, \tag{5}$$

- maximal:

$$R_{T\max} = R_{\min\max} + \Delta Q_T > R_T. \tag{6}$$

2.2. Time of the exhaustion of the stock

In the aspect of the time of the exhaustion of the stock we can also analyze limit values which can be based on the next expression:

$$tg\alpha = \frac{T_0}{R_T}. \tag{7}$$

Transforming equation (7) and substitute the average value of the angle of the exhaustion:

$$\overline{T_0} = R_T \cdot tg\overline{\alpha}. \tag{8}$$

To analyze the average time of the exhaustion it have to be determined

- a minimal value (T_0^A) and
- a maximal value (T_0^F) for the time of the exhaustion.

Depend on the relation between the average and the limit values of the time of the exhaustion we have to determine two different cases in the aspect of uploading levels:

- if $\overline{T_0} > T_0^F$, then the loading level requires reduction because in any cases the average stock level can be too high which will cause sufficient increasing in cost and storing capacity,
- if $\overline{T_0} < T_0^A$, then the loading level requires increasing because in any cases the intensity of the uploading process can be too high which will cause sufficient increasing in logistic tasks.

The minimal (T_0^A) and maximal (T_0^F) limits of the time of the exhaustion have to be determined to fit to the available storing and logistic capacities.

2.3. Correction of the loading quantities

If the uploading levels can not be changed there is another possibility to correct the uploading quantity. In this case ($R_T = const.$) the corrected uploading quantities can be calculated

- by $\alpha_{\min}$:

$$\Delta Q_{T\max} = \left(R_T - R_J\right) + \Delta R_{M1} = R_T - R_J + \frac{t_R}{tg\alpha_{\min}}, \tag{9}$$

where

$$tg\alpha_{\min} = \frac{t_R}{\Delta R_{M1}}, \tag{10}$$

- by $\alpha_{\max}$:

$$Q_{T\min} = \left(R_T - R_J\right) + \frac{t_R}{tg\alpha_{\max}}, \tag{11}$$

where

$$tg\alpha_{\max} = \frac{t_R}{\Delta R_{M2}}. \tag{12}$$

If $t_R = const.$, the required minimal stock limit:

$$R_J \rightarrow R_{\min\min} \geq 0, \tag{13}$$

and

$$R_{J\min} = 0.$$

Based on Figure 2.

$$tg\alpha_{\min} = \frac{t_R}{R_J}, \tag{14}$$

and

$$R_{J\min} = \frac{t_R}{tg\alpha_{\min}}. \tag{15}$$

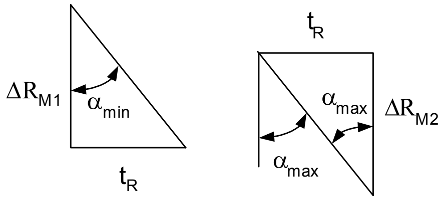

Figure 2. Relations for the limit values of the angle of the exhaustion
and the minimal stock limit

Based on the equations (2) and (3), if the value of $\Delta\alpha$ is high at constant uploading quantity we have to take

- very low uploading levels ($\Delta\alpha < 0$) or
- very high uploading levels ($\Delta\alpha > 0$) into consideration.

If we use an uploading strategy with changing quantities it is not enough to take only the minimal stock limit into account, but we have to take care on the angle of exhaustion of the stock and its changing.

2.4. Analysis of the stock level-time functions of element stores

At constant uploading terms the stock levels of the element store have to be suited to the distribution requirements of the finished products. Determination of the required stock levels based on Figure 3.

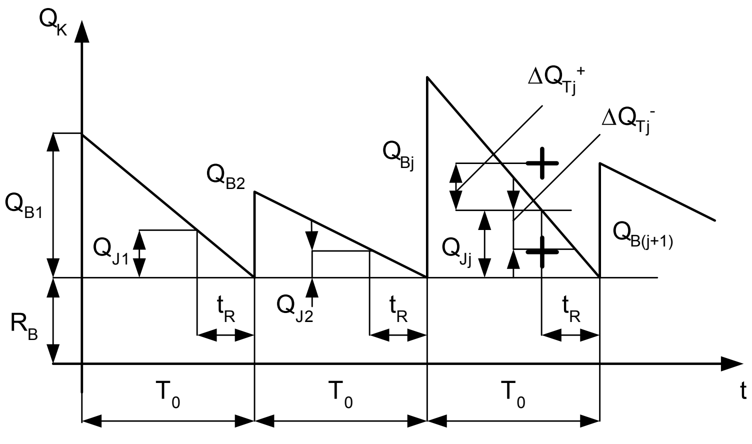

Figure 3. Example for an element supply function ($t_R \le T_0$)

Based on Figure 3. the prognostic value of the uploading when reach the minimal stock limit in term j can be calculated as:

$$Q_{Jj} = Q_{Bj} \frac{T_R}{T_0} . \tag{16}$$

The real exhaustion until term j can be

- if $Q_{Fj} = Q_{Jj}$, then the real exhaustion is equal to the prognostic value,

- if $Q_{Fj} > Q_{Jj}$ $\quad \Delta Q^{-}_{Tj} = Q_{Fj} - Q_{Jj}$, then the real exhaustion is higher then the prognostic value,
- if $Q_{Fj} < Q_{Jj}$ $\quad \Delta Q^{+}_{Tj} = Q_{Jj} - Q_{Fj}$, then the real exhaustion is lower then the prognostic value.

The safety stock level can be calculated with two different method:

a) $$R_{B1} = Max\left\{\Delta \overline{Q}_{Fj}\right\}, \tag{17}$$

b) $$R_{B2} = \overline{\Delta Q}^{-}_{Fj} + k\delta_{\Delta QF}. \tag{18}$$

where

$\overline{\Delta Q}^{-}_{Fj}$ the average value,

$\delta_{\Delta QF}$ the distribution value,

k an integer value (k=1, 2, 3, etc.) which takes the effect of the distribution into consideration.

Required uploading level required by the supplier at consignment stores

$$R_{T1} = \frac{T_0}{t_R}\left[Max\left\{Q_{Fj}\right\}\right], \tag{19}$$

or

$$R_{T2} = \frac{T_0}{t_R}\left[\overline{Q}_F + k\delta_{Fj}\right]. \tag{20}$$

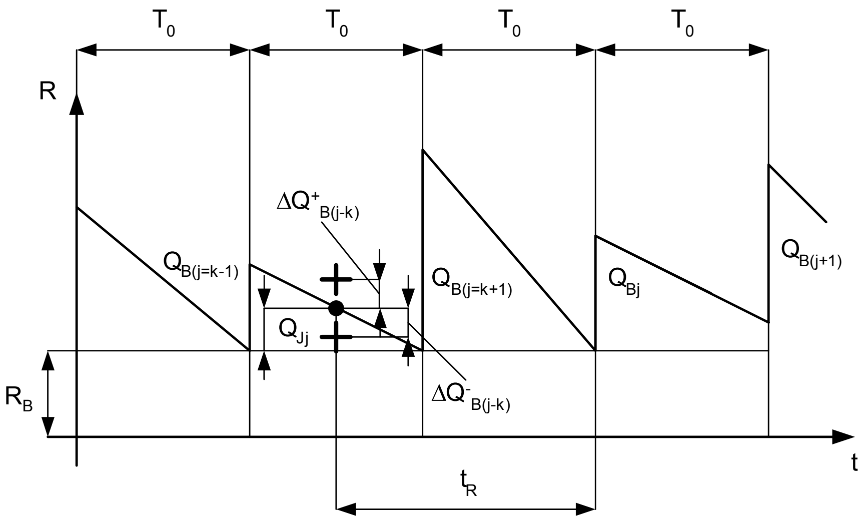

Figure 4. Example for the stock level of an element store ($t_R > T_0$)

To control the uploading process the uploading parameters have to be checked in determined terms, which are

- $\underline{Q}_{J(j-k)}$ the required stock level, depend on the actual production orders,
- $\underline{Q}_{F(j-k)}$ the real exhaustion (suited to the production orders),
- $\Delta Q^{-}_{T(j-k)} = Q_{F(j-k)} - Q_{J(j-k)}$ the overexhaustion,
- $\Delta Q^{+}_{T(j-k)} = Q_{J(j-k)} - Q_{F(j-k)}$ the underexhaustion.

The quantity of the elements to order can be calculated

- if $Q_{F(j-k)} = Q_{J(j-k)}$, then

$$Q_{T(j-k)} = \sum_{\varepsilon=j-k+1}^{j-1} \nu_{\varepsilon} Q_{B\varepsilon}, \tag{21}$$

where

ν_{ε} coefficient for taking the uncertainty of the order into account (has to be calculated by the historic data), its value can be

- if the real exhaustion is lower than the prognostic:

$$\nu_{\varepsilon} < 1,$$

- if the real exhaustion is higher than the prognostic:

$$\nu_{\varepsilon} > 1.$$

- if $Q_{F(j-k)} > Q_{J(j-k)}$, then

$$Q_{T(j-k)} = \sum_{\varepsilon=j-k+1}^{j+1} \nu_{\varepsilon} Q_{B\varepsilon} + \Delta Q^{-}_{T(j-k)}, \tag{22}$$

- if $Q_{F(j-k)} < Q_{J(j-k)}$, then

$$Q_{T(j-k)} = \sum_{\varepsilon=j-k+1}^{j+1} \nu_{\varepsilon} Q_{B\varepsilon} - \Delta Q^{+}_{T(j-k)}, \tag{23}$$

The safety stock level

$$R_{B1} = Max\left\{\Delta Q^{-}_{Tj}\right\} + \sum Q^{-}_{B\max}, \tag{24}$$

where

$\sum Q^{-}_{B\max}$ the fault of the evaluation during term $\underline{k}T_0$.

At consignment stores in generally the fulfillment of the statement $t_R > T_0$ is not required (Figure 4.).

3. Example for analysis of stock levels of a consignment store

The theoretical method was tested at a given product type of the consignment store of a Hungarian company named Schefenacker Automotive Parts Ungarn.

The stock level-time function of the given product can be seen on Figure 5. The Figure shows that the stock level is not constant and the changing is not periodic so the function can not be examined by methods designed for deterministic functions. If we create the angles of exhaustion from this function and compare them to the theoretical values calculated by the above mentioned method for the analyzed term then we get a new diagram can be seen on Figure 6.

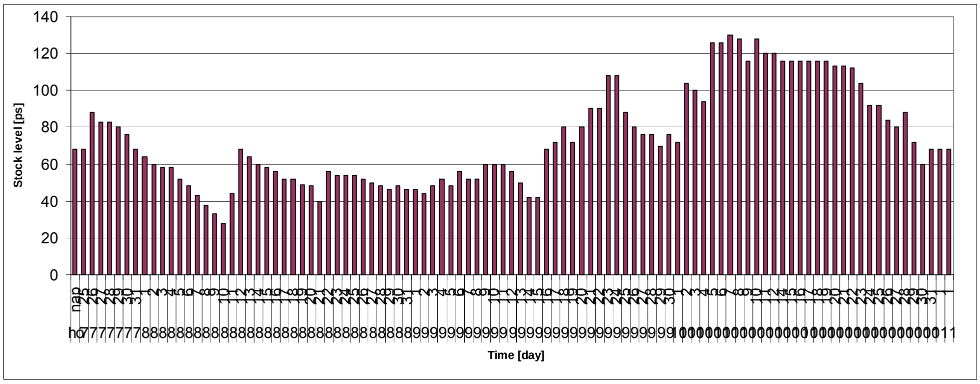

Figure 5. Stock level of the consignment store of the selected product type

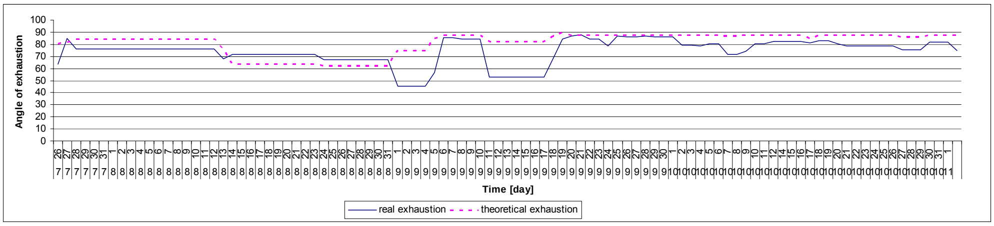

Figure 6. Angles of exhaustion of the product in the consignment store

By the analysis of the real exhaustion we get real angles of the exhaustion for the process which are suitable to determine a theoretical angle of the exhaustion to approach the real values. If we use this theoretical value to check the real exhaustion then functions on Figure 7. can be created where theoretical results are based on historic data.

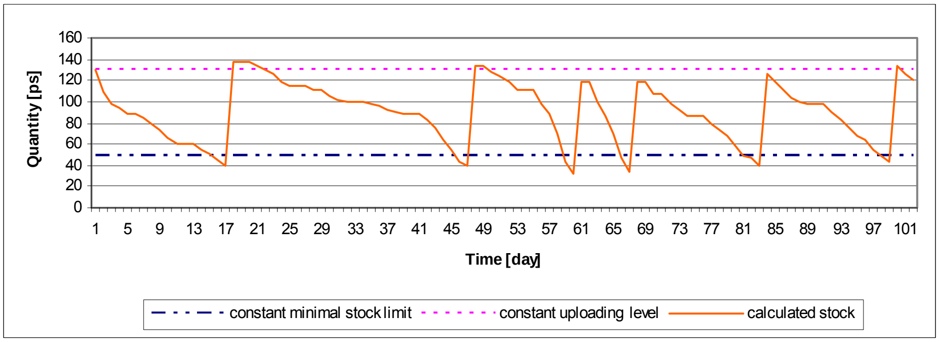

Figure 7. Effect of the theoretical angle of the exhaustion

Figure 8. shows the stock level of the selected product type in the consignment store using the theoretical angles and the real angles of exhaustion of the product.

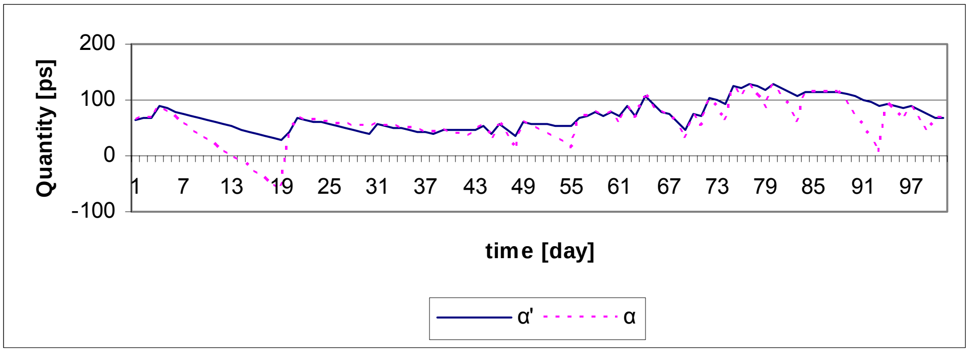

Figure 8. Comparison of the theoretical and the real stock level
(α' - theoretical, α - real)

To examine the effects of the theoretical exhaustion the next parameters have to be used

- average value of the stock levels,
- fulfillability of the continuous supplying,
- frequency of the supplying process.

Comparison of the average value of the stock levels in the theoretical and the real process can be seen on Figure 9. Comparison says that the amount of pieces stored in stock is higher about 20% in the theoretical situation than in the real process.

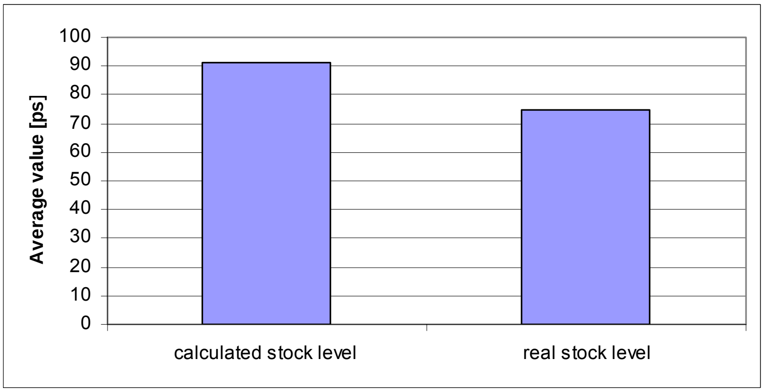

Figure 9. Average stock levels

In the aspect of the continuity of the supplying process there is only one case when the stock level of theoretical process goes under zero, in the other cases the method gives suitable results.

In the aspect of the intensity of the supplying process there is no significant difference between the real and the theoretical processes.

Summarizing the experiences in our example we can see that some parameter of the stocking process worse using the evaluated exhaustion than at the real situation, but this method let us to design the stocking process of consignment stores giving suitable results. As in our example we used the average value of the angles of exhaustion to calculate the evaluated exhaustion, to increase the precisity of the results (approach the optimal value) we can use the distribution of the angles of exhaustion in the further process.

4. Summary

Optimization of the operation of a consignment store is not an easy task in general. The main problem is the stochastic characterization of the exhaustion of the products which makes the using of deterministic methods impossible. In our research project we developed a theoretical method which uses historic data of a previous term to determine the optimal values for the parameters of the consignment storing process.
Main advantage of the researched method is to give possibility for the designers and operators of the consignment storing process to find (or at least approach) the optimal parameter values which is very difficult by any other way. As there can be seen in our example some results of the method contains mistakes. However this problem can be reduced by additional analysis which is the part of our further research activity.

References

[1] **Autóipari beszállítók szerelősorai ütemezésének és készletezésének logisztikai szempontú optimalizálása**, ME-AKKK research report, Miskolci Egyetem, 2005.
[2] DR. CSELÉNYI J. – DR. ILLÉS B. (szerk.): **Logisztikai rendszerek I**., Miskolci Egyetemi Kiadó, 2004.
[3] DR. CSELÉNYI J. – DR. ILLÉS B. (szerk.): **Anyagáramlási rendszerek tervezése I.,** Miskolci Egyetemi Kiadó, 2006. ISBN 963 661 672 8