MODIFICATION OF ASSIGNMENT OF FINAL PRODUCT REQUIREMENTS OF END USERS TO THE ASSEMBLY PLANTS BY DIRECT DISTRIBUTION AND COMPLEX OBJECTIVE FUNCTION IN A NETWORK-LIKE OPERATING ASSEMBLY SYSTEM

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Abstract: The events that led up to this scientific work that the detailed in the former publications application of assignment algorithms of assembly plants to the final product requirements of the end users in a cooperative assembly system we take simplified cost functions into account by the determination of the objective function. Leaning on the before-described model this work details the solution of distribution tasks by the help of complex objective function.

Keywords: Assignment, logistics, network, objective function, optimisation

1. Introduction

The network-like operating logistics integrated assembly system means when the production planning is planned integrated by the purchasing and distribution logistics system accordingly we search aggregate optimum of not merely the production but also the logistics resources and factors. The network-like means that the same product can be assembled by several assembly plants in different points and the components needful to assembling can be purchased from several different sited suppliers. Additionally the network-like means, that the procurement of components and the distribution of final products may be direct and indirect, which means, it happens by the help of distribution warehouses. In case of the network-like operating systems the logistics integrated production planning details how to search the optimal result having regarded to capacity-limits and conditions, and to fulfil the requirements of the end users according to described objective functions.

The optimal operation of this complex and largely cooperative logistics system requires an absolutely modern theoretical establishment of planning and control methods. The task to be completed is the logistics integrated assembly scheduling task, which includes the distribution and storage of final products and the storage of components. Different objective functions and conditions should be taken into consideration during the solution of these tasks. In the first case the cost function was chosen as the objective function, the components of which were detailed in [3]. The optimisation was completed by a multistage heuristic method jointed each other as a hierarchically organized feedback The authors worked out this solution because high number of cost function parameters are to optimise. The modules

of a multistage optimisation are illustrated in [4]. The principles, solution methods and heuristic algorithm of the assignment are demonstrated in [5]. This paper determines the assignment by the help of a complex objective function.

1.1 Total cost function of the model

$$C = C_P + C_T + C_W + C_A + C_{AP} + C_\Psi + C_S + C_D \to min. \quad (1)$$

which can be obtained as a sum of the following costs: purchase costs of components (C_P), transportation costs of components (C_T), warehousing charges of components (C_W), assembly costs (C_A), changeover costs of assembly lines (C_{AP}), costs of standby of lines (C_ψ), storage costs of final products (C_S) and delivery costs of products (C_D).

In the former case as we used to for the determination of the annual amount of the final products of the individual user we simplified the total cost function (1) and then only the assembly and delivery costs should be considered. Because that module considered the schedule of assembly and transportation, the warehousing cost of the components and final products could not be taken into consideration and the considered costs were also global and simplified. The above-mentioned cost-components had not to be taken into account by optimisation, because these components were not known by that step of assignment, but we take these into account in this module and effects of these components appear from the principle of feedback.

1.2 Simplified objective function of the assignment in case of product k

$$C_I^k = C_A^k + C_D^k \to min. \quad (2)$$

where C_D^k is the delivery cost, C_A^k is the assembly cost.

The matrix **Q** gives the annual quantity ordered from product k by the user μ. The searched matrix **Y** shows that

- The user μ obtains the product k from the assembly plant λ or not. The $y_{\mu\lambda}^k$ value is 0

  if not, or 1 if yes (case **a**) with the following condition: $\sum_{\lambda=1}^{n} y_{\mu\lambda}^k = 1$. $\quad (3)$

- Or how much part of the final product k will be come out to the end user μ from the

  assembly plant λ (case **b**). Conditions are: $0 \le y_{\mu\lambda}^k \le 1$ and $\sum_{\lambda=1}^{n} y_{\mu\lambda}^k = 1$. $\quad (4)$

1.2.1 Delivery cost in case of product k

$$C_D^k = \sum_{\lambda=1}^{n} \sum_{\mu=1}^{w} c_D^k Q_\mu^k y_{\mu\lambda}^k s_{\mu\lambda} \quad (5)$$

where c_D^k is the specific delivery cost of product k, $s_{\mu\lambda}$ is the length of delivery route.

1.2.2 Assembly cost in case of product k

$$C_A^k = \sum_{\lambda=1}^{n} \sum_{\mu=1}^{w} Q_\mu^k y_{\mu\lambda}^k c_{A\lambda}^k \qquad (6)$$

where $c_{A\lambda}^k$ is the specific assembly cost in case of product k in the assembly plant λ.

1.2.3 Simplified cost function of the assignment in case of product k

The objective function (2) becomes the following formula by the considered and simplified objective functions [4]:

$$C_1^k = \sum_{\lambda=1}^{n} \sum_{\mu=1}^{w} Q_\mu^k y_{\mu\lambda}^k (c_D^k s_{\mu\lambda} + c_{A\lambda}^k) \rightarrow min. \qquad (7)$$

In chapter 2 (below) we analyse how the assignment changes in light of the complex objective function if we depart from the simplified cost function.

2. Assignment of users to plants by complex objective function

In the first step we determine the starting assignment in accordance with simplified objective function and the second step follows it which is a logistics integrated assembly scheduling.

$$C_{AP\lambda} + C_{S\lambda} = C_{\lambda 0} \rightarrow min. \qquad (8)$$

where $C_{AP\lambda}$ – assembly preparation cost, $C_{S\lambda}$ – storage cost, λ – assembly plant.

2.1 Cost functions
2.1.1 Assembly preparation cost

$$C_{AP\lambda} = \sum_{k=1}^{g} c_{P\lambda k}^A z_{\lambda k} \qquad (9)$$

where $c_{P\lambda k}^A = [EURO / cycle]$ is the preparation cost of a serial (product k) in the plant λ;
$z_{\lambda k}$ is the number of serials (product k) in the plant λ during the program time.

2.1.2 Storage cost

$$C_{S\lambda} = \sum_{k=1}^{g} c_{S\lambda k} \sum_{j=1}^{v^*} A_{\lambda k j} \qquad (10)$$

where $A_{\lambda k j}$ is the area of stock diagram by product k in case of cycle j in the plant λ;
$c_{S\lambda k} = [EURO/p.hour]$ is the specific storage cost;
v^* is the number of cycle in program time.

2.1.3 Loss cost from standby of assembly lines

$$C_{\Psi\lambda}^S = c_{\Psi\lambda} \sum_{j=1}^{v^*} \sum_{\delta=1}^{p_\lambda} \tau_{\lambda\delta j} \qquad (11)$$

where $k_{\Psi\lambda}$=[EURO/hour] is the loss cost from specific idle time (idle time periods).

$\tau_{\lambda\delta j}$ is the idle time of assembly line δ in case of cycle j in the assembly plant λ (do not occur assembly, assembly preparation and maintenance).

2.1.4 Loss cost from late-delivery

$$C_{\Psi\lambda}^{L} = \sum_{\mu=1}^{w}\sum_{k=1}^{g} c_{\Psi\lambda\mu k} \sum_{j=1}^{v^*} y_{\lambda\mu kj}\Delta T_{\lambda\mu kj} \tag{12}$$

where $c_{\Psi\lambda\mu k}$=[EURO/p.hour] is the specific loss cost from late delivery of final product k from assembly plant λ to the end user μ;

$y_{\lambda\mu kj}$ is the number of delivered product k in cycle j from plant λ to the user μ;

$\Delta T_{\lambda\mu kj}$ is the delay time of late-delivered final product k in cycle j from assembly plant λ to the end user μ.

3. Modification of the assignment

Otpimization is accomplished by a concrete example, where basic data (are stated in the former publications) are the followings: *n=3,* w*=6, g=8*. Values of the ordering matrix Q = [$q_{k\mu}$] can be between 1000 and 6000, the average of these values is about 2000 pieces. The capacity matrix A can be taken in like manner.

$$
Q = k
\begin{array}{c}
1 \\ \vdots \\ \\ \\ \\ \vdots \\ \\ g
\end{array}
\begin{bmatrix}
0.5 & 2.5 & 0 & 0 & 1.5 & 1 \\
0 & 1 & 2 & 3 & 2 & 0 \\
3 & 0 & 1 & 0.5 & 0 & 0 \\
1.5 & 0 & 0 & 1 & 2.5 & 0 \\
2 & 1.5 & 1 & 0 & 3 & 0 \\
0 & 0 & 2 & 0 & 0 & 3 \\
0 & 0.5 & 0 & 0 & 1 & 2 \\
1 & 0 & 3 & 0 & 0.5 & 0
\end{bmatrix}
[2000\,pieces]
$$

columns headed 1 … μ … w

$$
A = k
\begin{bmatrix}
1.5 & 1 & 0.5 \\
0 & 2 & 2 \\
0.5 & 0.5 & 1.5 \\
1 & 1.5 & 0 \\
1.5 & 0 & 2.5 \\
0.5 & 1 & 1 \\
0 & 1 & 1 \\
1.5 & 0 & 1
\end{bmatrix}
[4000\,pieces]
$$

columns headed 1... λ ...n

$$
S = \mu
\begin{bmatrix}
0.2 & 0.8 & 1.5 \\
2.5 & 0.6 & 1.2 \\
1.8 & 2 & 1 \\
0.6 & 0.5 & 1.5 \\
2 & 1 & 2.5 \\
2.2 & 1.2 & 0.2
\end{bmatrix}
[100km]
$$

columns headed 1... λ ...n

The values of the route matrix S = [$s_{\mu\lambda}$] (between assembly plants and end users) can change between 20 and 250 km, the average value of elements is about 100 km. Values of the specific delivery and assembly cost as well as time are taken in harmony with the former example.

$$
C^{D} = k
\begin{bmatrix}
0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \\ 1 \\ 1.1 \\ 1.2
\end{bmatrix}
c_o\delta\left[\frac{EUR/piece}{100km}\right]
$$

$$
C^{A} = k
\begin{bmatrix}
0.7 & 0.525 & 0.875 \\
1 & 0.75 & 1.25 \\
1.2 & 0.9 & 1.5 \\
0.8 & 0.6 & 1 \\
1.3 & 0.975 & 1.625 \\
0.9 & 0.675 & 1.125 \\
1.4 & 1.05 & 1.75 \\
1.1 & 0.825 & 1.375
\end{bmatrix}
c_o\left[\frac{EUR}{piece}\right]
$$

columns headed 1... λ ...n

$$
t^{A} = k
\begin{bmatrix}
0.2 & 0.3 & 0.1 \\
0.8 & 0.5 & 0.4 \\
0.6 & 1 & 0.8 \\
0.9 & 0.7 & 0.8 \\
1 & 0.2 & 0.4 \\
0.5 & 0.1 & 0.3 \\
0.7 & 0.6 & 0.5 \\
0.4 & 0.2 & 0.3
\end{bmatrix}
\left[\frac{hour}{piece}\right]
$$

columns headed 1... λ ...n

Instead of c_0 the notation of c_{0x} may be required where x means the index of specific costs according to the before-defined $c_{0x} = c_x^* c_0$ (where c_x^* is the proportional parameter). By this solution it is a chance to modify the ratio of the specific costs. In this instance c_{0x} equals c_0 in case of every „x", i.e. $c_x^* = 1$, so the c_0 specific basic cost is the same by every cost function. Specific assembly preparation cost and time matrix, as well as storage cost matrix:

$$
c_{P\lambda k}^A = k
\begin{bmatrix}
2 & 1.5 & 4 \\
3 & 3.2 & 1.6 \\
1.5 & 2.8 & 2.2 \\
4 & 2.6 & 1.8 \\
2.5 & 1.7 & 3.2 \\
3.5 & 2.4 & 2.5 \\
1 & 3 & 1.3 \\
1.2 & 3.2 & 3.4
\end{bmatrix}
c_0 \left[\frac{EUR}{cycle} \right]
\quad
t_{P\lambda k}^A = k
\begin{bmatrix}
0.9 & 1.1 & 0.7 \\
1.2 & 0.8 & 1 \\
0.6 & 0.9 & 1.1 \\
1.4 & 0.7 & 0.8 \\
1.3 & 1 & 0.7 \\
1 & 0.6 & 1.2 \\
0.8 & 0.9 & 0.6 \\
1.2 & 1 & 0.7
\end{bmatrix}
[hour]
\quad
c_{\lambda k}^S = k
\begin{bmatrix}
0.4 & 0.4 & 0.6 \\
0.2 & 0.3 & 0.5 \\
1 & 0.7 & 0.6 \\
0.5 & 0.4 & 1.1 \\
0.2 & 0.3 & 0.8 \\
0.9 & 0.5 & 0.6 \\
0.7 & 0.3 & 0.8 \\
1 & 1.1 & 0.9
\end{bmatrix}
\frac{c_0}{10} \left[\frac{EUR}{p.hour} \right]
$$

with row indices $1 \ldots \lambda \ldots n$ as columns and $1, \vdots, 1.5, 4, 2.5, \vdots, 1, g$ as rows.

Loss cost matrix from standby of assembly lines: $c_{\Psi\lambda}^S = \dfrac{c_0}{100} \begin{bmatrix} 0.6 & 0.7 & 0.8 \end{bmatrix} [EUR / hour]$ with columns $1 \ldots \lambda \ldots n$.

We defined a data-structure to be capable to examine by sensitivity analysis and to compare by different optimisation methods. By the solution with complex objective function of the before-mentioned example – is stated in the former publications too – we take the assignment matrix Y respecting the case **b** (4) is effected by the Hungarian Method to the starting data, so supposing that the assembly plants are assigned to the end users. Taking the complex objective function into consideration by the help with the before-described algorithm the assignment change as follows:

$$
Y = \begin{matrix} 1 \\ \vdots \\ \mu \\ \vdots \\ w \end{matrix}
\begin{bmatrix}
1^{0^0} & 0^{0^0} & 0.33^{0^{0.67}} & 1^{0^0} & 1^{0^0} & 0^{0^0} & 0^{0^0} & 1^{0^0} \\
0^{0.8^{0.2}} & 0^{0^1} & 0^{0^0} & 0^{0^0} & 0^{0^1} & 0^{0^0} & 0^{1^0} & 0^{0^0} \\
0^{0^0} & 0^{0^1} & 0^{0^1} & 0^{0^0} & 0^{0^1} & 0.5^{0.5^0} & 0^{0^0} & 0.50^{0^{0.50}} \\
0^{0^0} & 0^{1^0} & 0^{1^0} & 0.5^{0.5^0} & 0^{0^0} & 0^{0^0} & 0^{0^0} & 0^{0^0} \\
1^{0^0} & 0^{0.5^{0.5}} & 0^{0^0} & 0^{1^0} & 0.17^{0^{0.83}} & 0^{0^0} & 0^{1^0} & 1^{0^0} \\
0.5^{0^{0.5}} & 0^{0^0} & 0^{0^0} & 0^{0^0} & 0^{0^0} & 0^{0.33^{0.67}} & 0^{25^{75}} & 0^{0^0}
\end{bmatrix}
$$

with column headers $1 \ldots k \ldots g$.

The three-dimensional matrix Y is converted in the interest of the briefer representation, that $Y = [y_{\mu\lambda}^k]$, so the matrix $y_{\mu k}$ can be seen in the plane, i.e. the rows of the matrix mean the end users, the columns mean the final products and the values λ are represented with smaller numbers. It can be seen from the assignment matrix Y which final products will the assembly plants deliver to the end users by the optimisation respecting the complex cost functions.

This matrix Y - which contains the annual volume of final products in relation to assembly plant and end users - has to be divided in 250 cycles (daily delivery). In this paper we divide equal cycles.

After solving the example the next three figures demonstrate the availability of the assembly lines of the plants. In this figures the empty rectangles mean the preparing times, the colours represent the assembly times of products while the straight lines mean the idle times of assembly lines (Vertical axe: each assembly line is shown; horizontal axe: time)
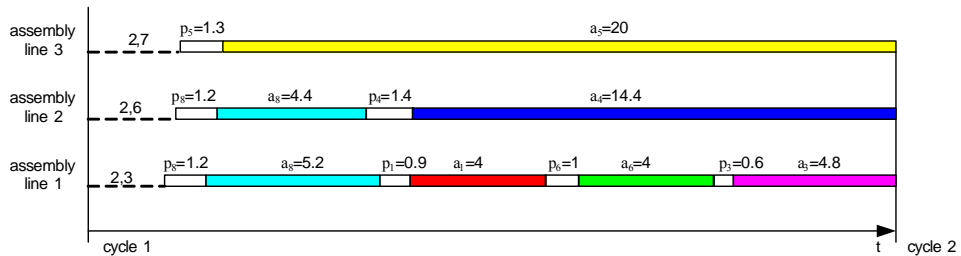

Figure 1. Chart of assembly lines of plant 1

Readers can conclude from Figure 1 that the three assembly lines work with similar capacities. Showing the figures describe below:

Plant 1	capacity	working time	
Line 1	90.42 %	21.7 hours	
Line 2	89.17 %	21.4 hours	
Line 3	89.75 %	21.3 hours	
Total	89.44 %		

In order to perform a minimal number of working lines, series with No. 8 product are to divide. See Figure 2.
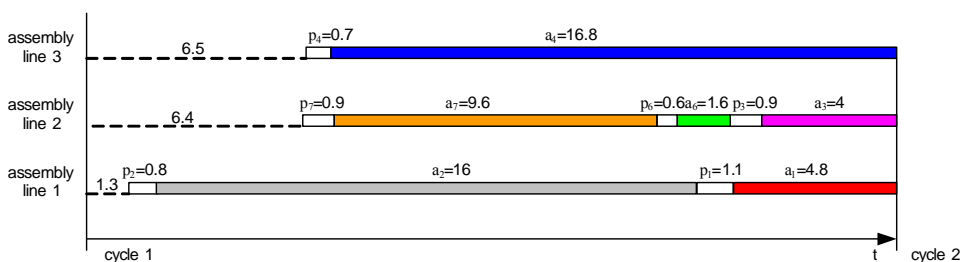

Figure 2. Chart of assembly lines of plant 2

Examining the three assembly lines of plant 2, readers can see, none of them achieves 100 % capacity.

Plant 2	capacity	working time	
Line 1	94.583 %	22.7 hours	this is the best
Line 2	73.33 %	17.6 hours	low capacity
Line 3	72.916 %	17.5 hours	low capacity
Total	80.27 %		worst of the three plants
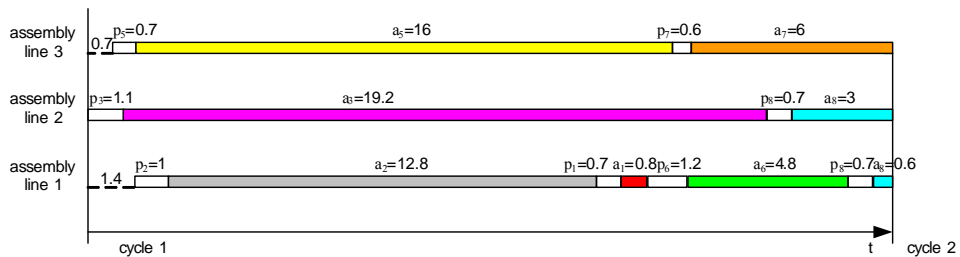
Figure 3. Chart of assembly lines of plant 3

Line 2 capacity works 100 %.

Plant 3	capacity	working time	
Line 1	94.16 %	22.6 hours	the worst capacity
Line 3	97.083 %	23.3 hours	
Total	97.083 %		best of the three plants

In order to work with the minimal numbers of assembly lines, the series with the belonging No. 8 product are also to divide.

The storage costs of final products can be easily stated by the help of figures. Using the specific costs given in chapter 2, the searched values of the objective function can be calculated too.

Table 1. Values of delivery and assembly cost per plant and user

User	delivery cost [c_0]				assembly cost [c_0]			
	plant 1	plant 2	plant 3	total	plant 1	plant 2	plant 3	total
1.	2260	0	4800	7060	12900	0	6000	18900
2.	-	2100	6000	8100	-	3150	8250	11400
3.	10080	4000	10000	24080	5100	1350	15375	21825
4.	540	2950	0	3490	800	6000	0	6800
5.	8000	8100	16000	32100	4500	6600	10625	21725
6.	1320	3720	1580	6620	700	2400	10625	13725
Sum	22200	20870	38380	81450	24000	19500	50875	94375

Analysing the delivery costs the lowest value (3490 c_0) can be found by the end user 4 while the largest delivery cost is realised by the end user 5 (16000 c_0). In case of the assembly costs the least value (6800 c_0) is also established by the end user 4 while the largest cost is stated by the user 3 (21825 c_0) which is followed close by the user 5 (21725 c_0). In case of both delivery and assembly cost the largest proportion (47,12 % and 53,91 %) arises by the assembly plant 3 in respect to sum-total costs while same proportions in case of assembly plant 1 are 27,26 % and 25,47 % as well as the least percentage of sum-total costs (25,62 % and 20,66 %) can be read by the assembly plant 2.

Table 2. Values of preparation and storage costs per plant and user

User	preparation cost [c_0]				storage cost [c_0]			
	plant 1	plant 2	plant 3	total	plant 1	plant 2	plant 3	total
1.	1427	0	366,67	1793,67	6014,917	0	532	6546,917
2.	-	562,5	840	1402,5	-	741	2420	3161
3.	1175	300	2243,33	3718,33	1870,375	570	3949	6389,375
4.	250	1300,43	0	1550,43	360	3078	0	3438
5.	400,5	577,17	500	1477,67	2567,458	3228	2250	8045,458
6.	100	487,5	1450	2037,5	496	927	2070	3493
Sum	3352,5	3227,6	5400	11980,1	11308,75	8544	11221	31073,75

Analysing the assembly preparation costs the least value can be stated by the end user 2 (1402,5 c_0) which is followed by the user 5 and 4, at the same time the largest preparation cost (3718,33 c_0) can be found by the end user 3. In case of the storage costs the lowest value (3161 c_0) can be also established by the end user 2 which is followed by the user 4 and 6, while the largest cost (8045,458 c_0) arises by the user 5. The assembly plants 1 and 3 represent the largest proportion (36,39 % and 36,11 %) of the storage costs while the second plant has the least proportion (27,5 %).
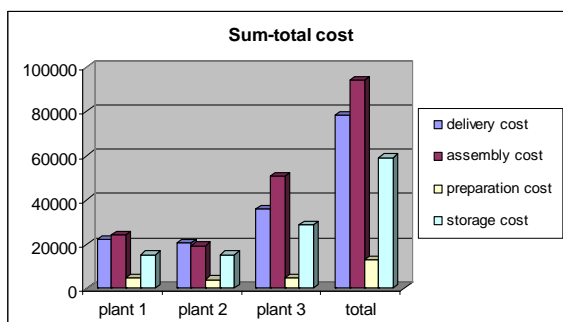

Figure 4. Diagram of sum-total cost per plant and component

The storage costs decreased but the delivery costs increased in each plant. The assembly cost increased in plant 2, it is constant in case of plant 3 while it decreased in plant 1. The assembly preparation costs decreased by plant 1 and 2, but increased in plant 3.
Sum of loss costs from standby of the assembly lines is 77,1 c_0, which comes in case of every plant as follows: 24,7 c_0 + 34,8 c_0 + 17,6 c_0. It can be seen that the assembly plant 2 comes up in the largest proportion (45,1 %) which is unsurprising after all the availability of assembly lines of this plant was the worst (80,27 %). Loss cost from late-delivery is zero in this example, because even distribution (each 250th of the annual production per cycle) is supposed.

Table 3. Assignment costs per product

Products	costs [c_0]				
	delivery	assembly	preparation	storage	total
1.	7320	7350	1875	3788	20333
2.	11480	16000	1200	9416	38096
3.	7120	12300	1625	1518	22563
4.	6030	6800	1002,6	3456	17288,6
5.	20900	22750	802,5	5960	50412,5
6.	10800	9000	2100	3360	25260
7.	4840	9450	1075	2148	17513
8.	12960	10725	2300	1427,75	27412,75
Sum-total	81450	94375	11980,1	31073,75	218878,85

Readers can trace from Table 3 that product **No. 4** has the <u>lowest</u> ***assembly cost*** (6800 c_0). <u>Highest cost</u> of assembling can be seen in case of product **No. 5** (22750 c_0).
Regarding the ***delivery cost*** – <u>smallest price</u> in case of **No. 7** (4840 c_0), and product **No. 5** shows the <u>highest price</u> (20900 c_0).
As for the ***storing cost*** – <u>smallest value</u> seen in case of product **No. 8** (1427,75 c_0). <u>Highest value</u> is product **No. 2** (9416 c_0).

Summarized cost: <u>smallest price</u>:   product **No. 4** tightly followed by product **No. 7**.
                                        <u>highest price</u>:   product **No. 5** followed with a considerable difference by product **No. 2**.

Examining the table-values, <u>the smallest cost-rate is shown</u> in the *assembly preparation* (5.5 %) comparing with *storing* (14.2 %), *transporting* (37.2 %) and *assembling* (43.1 %) costs. *Transporting* cost in case of product No. 6 and product No. 8 means the highest rate of the summarized cost. In all other cases *assembly* means a big part of the summarized cost.

Regarding the summarized cost
- transporting cost grew by 3.67 %;
- assembling cost shows hardly any increase (0.05 %);
- assembly-preparing and storing costs fell with a considerable rate (8.02 %; 38.37 %);
- summarized cost increased by 7.43 %.

Product No. 2, No. 7 and No. 8 show increase in case of the summarized cost. The other products show decrease.

4. Summary

Possibilities of assignment by the help of the complex objective function are described and the algorithm of logistics integrated assembly scheduling is demonstrated in this scientific paper. Finally come to the optimum sensitivity analysis of the worked out algorithm for optimisation with the complex objective function and the analysis of proportion of cost components.

5. Acknowledgements

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