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MATHEMATICS EDUCATION FOR LOGISTICS ENGINEERING

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Abstract: Mathematics is a crucial language in all engineering courses and researches where mathematical modeling, simulation and manipulation are commonly used. Engineering Mathematics courses are considered difficult courses in engineering curricula. This is reflected in engineering students' performance at the end of each semester for these courses. Our goal is to overview a few questions on mathematics as a core subject of engineering.

Keywords: generic skill, mathematical modelling, differential equation, solution techniques, catenary

Introduction

An engineer must have a good command of basic math skills. The goal of teaching mathematics to engineering students is to find the right balance between the practical application of mathematical equations and in-depth understanding of the living situation. Mathematics is a key language in all engineering courses and research where mathematical modeling, simulation and manipulation are commonly used. But Engineering Mathematics courses are considered difficult courses in engineering curricula. This is reflected in the performance of engineering students at the end of each semester for these courses. In all levels of education, the traditional method of teaching is lecture-based teaching. The description of a lecture is conducted by using the traditional method. Although this method is able to produce graduates and knowledge to the students, but the majority of students is superficial and focused on the development of test-passing capabilities.

In teaching Engineering Mathematics courses for engineering students, subsequently, found that these students encounter some difficulties and act indifferently toward these learning method. The students often regarded Engineering Mathematics courses as uninteresting and difficult in engineering curriculum. The problems are first year students' varying level of knowledge in mathematics and low participation in teaching. Other possible facts are motivation, ability to work independently and acclimatization to university studying environment. On the other hand, the impact of mathematical thinking skills on an engineer will enable them to use mathematics in their practice [1]. Attitudes of engineering undergraduates toward mathematics were studied by Miika *et al.* [2] and this provides us better understanding to engineering students' actual knowledge and the lack of knowledge in mathematics. It addresses long term needs for abilities and skills because these impose strong constraints on secondary and even on elementary education. The goal of our higher education is to produce competent engineers. To do this, the university faculty

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needs certain skills from high school to develop them.

Mathematics is a fundamental subject for all engineering courses and researches, where mathematical modeling, manipulation and simulation are used extensively. Current educational approaches do not provide enough functionality to bring it to life. Current introductions to problems have rather passive understanding as goals, rather than doing things with them. A more active approach could have successful in real life settings. An overview of structure and practice of mathematics is given in [3].

Precision and functionality: Mathematics has elaborated a set of explicit rules of reasoning. People often have trouble learning to use these rules effectively because rules in most other systems are sufficiently unclear and ineffective that precision is a waste of time, while in mathematics anything less than full precision is a waste of time.

Students should be taught to record their reasoning in a way that can be checked for errors. Criteria for good work formats are:

- record enough detail so reasoning can be constructed and checked for errors,
- be compact and straightforward,
- help to organize the work in ways consistent with human cognitive constraints.

The most important strategy is teacher diagnosis of errors that students can't find themselves. The wrong answers should be corrected. The student should explain his or her reasoning following his or her written work record. If the record is incorrect, steps were skipped or appropriate templates were not used, then the work should be redone. If the work record is appropriate, then it can be reviewed and mistakes quickly corrected.

Definitions and key statements: The important objects, terminology and properties should be given by brief and precise formulations:

- The formulations should be constructed primarily by professional mathematicians with education feedback.
- Students should memorize them, so they can be reproduced exactly. Definitions provide key points and functional understanding deepens around the definitions.
- Explanation of what definition means is best given after the definition, not before. Putting too much explanation first almost guarantees confusion.

Intuitions: A great deal of mathematical activity is guided by intuition. Intuitions developed by working with a good definition are often effective. These intuitions based on meaning of physical analogue are almost never satisfactory, are acceptable if passive understanding is the goal.

In practice it is not enough just to have good basic mathematics knowledge. An engineer is also required to have good generic skills such as a good communication skill, positive thinking and able to work independently. A mathematical model is developed to represent a physical phenomena and this model is analyzed and solved mathematically. A mathematical conclusion about the model is reliable when the connection between the physical situation and the model is appropriate.

Mathematics is a basic subject for students of engineering. Structuring the topics the goal is to teach useful methods for future engineers. For the future studies the methods and simple applications are essential. Considering the main objective of mathematical training at technical universities is to teach students to solve applied problems. Mathematical skills are used to describe and cope with a wide range of problems. These key skills are about understanding when particular techniques should be used, how to carry them out accurately and which techniques should be applied in particular situations.

Next a short introduction to the chapter of differential equations is represented together with an example to exhibit an approach to a real life problem.

1. The application of differential equations

Many natural laws give a relation between the rates of change of various quantities, rather than relation between the quantities themselves.

A rate of change of a quantity is represented by the so called derivative of this quantity. An equation involving derivatives is called a differential equation. The differential equations have had a great influence on the history of science and represent that mathematical methods are applied to the real world. We intend to show some of the uses of differential equations, firstly the origin and then the later applications in other fields of science [4], [5]. The subject of differential equations has its primary historical origin in Newtonian mechanics. His scientific work was based on differential equations, which he applied successfully to the study of Nature. For example, Newton's second law of motion (force)=(mass) (acceleration) is a differential equation. When combined with his law of gravitation, Newton's laws of motion enabled him to compute the orbit of the planets, the moon and comets. He estimated the weight and density of the sun and moon. When only the sun and a single planet are considered, the differential equation obtained by Newton is solvable in an elementary way. As a result Newton could give a brief derivation of the three laws of Kepler, which Kepler had observed through lifetime astronomical observations. The problem of determining the motion of two gravitating masses, such as the sun and a planet, is called two-body problem. The three body problem is to determine the motion of three gravitating masses, such as earth, sun and moon. This problem is not elementary at all. Many famous mathematicians haves been dealing with this problem since Newton.

Many excellent mathematicians have worked on differential equations. Some of the leading names are Leibniz, Daniel Bernoulli, Euler, Laplace, Lagrange, Fourier, Poincare, Picard, Liapunov and Volterra. These later researchers showed that differential equations can be applied not only to Newtonian mechanics, but to a high variety of scientific fields. For example, differential equations are applied to fluid flow, propagation of sound waves and the flow of electricity in a cable. Fourier's book (1822) entitled The Analytic Theory of Heat has been called the Bible of mathematical physics. By differential equations, Maxwell predicted the existence of electromagnetic waves (radio waves) before they were observed experimentally by Hertz. The theory of geometric optics depends on differential equations and so does the theory of deformation of elastic structures (beams, membranes, etc.). Chemical reactions, radioactive decay can be modeled also by differential equations.

These examples come from such fields as physics, chemistry, engineering which are sometimes called as the "exact sciences", where the relevant differential equation can be formulated easily.

There are other fields in which differential equations are also essential. Some of these topics are connected with economics, ecology and biology. For example, the successful program of eradicating smallpox is dedicated to Daniel Bernoulli, who set up differential equation for the progress of an infections disease in 1760. His work gave a scientific justification for the risk of vaccination. Another example when differential equations are used to study the fluctuations of animal populations, such as the Arctic fox or such as fish in the Adriatic. Both these examples have played a role in the development of mathematical ecology. Further examples can be given by the mathematics of heart physiology, the transmission equations for nerve impulses, the growth laws of tumors, etc.

To conclude this we remark that it would be a mistake to think that differential equations are important only in connection with other fields of science. The Mathematical Reviews publishes abstracts of more than 75 new papers on differential equations per month and the number has been increasing from year to year. If we want to describe a movement or process by differential equation first we need a mathematical model. The mathematical description of a phenomenon always entails some simplification. A more exact representation of the physical phenomena can be described by a more complicated differential equation which makes it possible to take additional factors into consideration. To give solution of some complicated problems or to characterize its properties are still open for the researchers.

2. An example: chain curve, catenary

Take a flexible chain of uniform linear mass density. Suspend it from the two ends.

What is the curve formed by the chain?

Galileo Galilei said that it was a parabola. This time Galileo was not correct. This curve is called a catenary. Let us determine the shape of the chain.

Let us denote by *P* an arbitrary point of the chain, and by *O* the point of the chain located at the origin of the Cartesian coordinate system. Moreover, let the low point of the chain be the origin. Take a part of a chain in the interval between *O* and *P*. Let us assume that the chain is in rest. That means that the net force on it is zero.

There are three forces acting on the chain interval *OP*. One force is the tension from the chain to the left. Call this force $\vec{H}$. It acts in a horizontal direction to the left. There is also a force from the chain on the right, exerted at point *P*. This force will be called $\vec{T}$. Its direction is what we need to determine, because that will tell us the slope of the chain at *P*. The third force is the weight on the chain, $\vec{W}$. Since this is a gravitational force, its direction can only be vertical and downward.

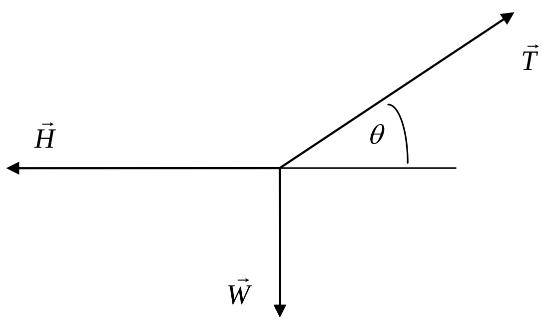

Figure 1. The forces

In the illustration, we can see the direction of $\vec{T}$, which gives us the slope at P. Force $\vec{T}$ has a horizontal component for which $|\vec{T}|\cos\theta = |\vec{H}|$ and for the vertical component $|\vec{T}|\sin\theta = |\vec{W}|$. That means:

$$\tan\theta = \frac{dy}{dx} = \frac{W}{H},$$

where W and H denote the magnitudes for $\vec{W}$ and $\vec{H}$, respectively, not vectors. The chain carries only its own weight. That weight is proportional to the length of the chain between O and P.

Let us denote by S the length of the chain, where $w\,[N/m]$ is the weight per unit length of the chain, then $W = ws$. Well, H is clearly a constant. It is the tension at point O, which is the same no matter what point is chosen for P. W is the weight on the chain between O and P. We show how can it be determined?

Again, the weight is proportional to the length of the chain between O and P. It cannot be computed by simply multiplying the horizontal distance by a constant. Hence we need to find the length of the chain between O and P.

Let y be the height of the chain. Again, w represents the linear weight density, it is weight per length of chain. If s denotes the length of the chain between O and P, this means that the weight of the chain between O and P must be ws.

You may recall this formula for the length of a curve on the interval (a,b):

$$\int_a^b \sqrt{1+y'^2}\,dx\,.$$

We can use it to find s, the curve length on the interval $(0,x)$:

$$s = \int_0^x \sqrt{1+y'^2}\,dt\,.$$

The derivative, dy/dx, is itself a function of s:

$$\frac{dy}{dx} = \frac{W}{H} = \frac{ws}{H} \text{ and } s = \frac{H}{w}\frac{dy}{dx}\,.$$

Make this substitution for s in the integration equation above:

$$\frac{H}{w}\frac{dy}{dx} = \int_0^x \sqrt{1+y'^2}\,dt, \qquad \frac{dy}{dx} = \frac{w}{H}\int_0^x \sqrt{1+y'^2}\,dt\,.$$

Differentiate both sides of the equation with respect to x:

$$\frac{d^2y}{dx^2} = \frac{w}{H}\sqrt{1+y'^2}, \quad \text{or} \quad y'' = \frac{w}{H}\sqrt{1+y'^2},$$

which is a second order nonlinear ordinary differential equation.

Substituting $y' = p(x)$ one gets the following first order differential equation

$$\frac{dp}{dx} = \frac{w}{H}\sqrt{1+p^2}$$

and separating the variables we obtain

$$\frac{dp}{\sqrt{1+p^2}} = \frac{w}{H}dx \, .$$

Let us take the integral of both sides

$$\int \frac{dp}{\sqrt{1+p^2}} = \int \frac{w}{H}dx \, ,$$

and apply the primitives to both sides

$$ar\sinh p = \frac{w}{H}x + C \, ,$$

with integration constant C . Hence

$$y' = p = \sinh(\frac{w}{H}x + C) \, ,$$

$$y = \int \sinh(\frac{w}{H}x + C)\, dx = \frac{H}{w}\cosh(\frac{w}{H}x + C) + D \, ,$$

which gives the general solution to the second order differential equation. In order to determine constants C and D we apply the initial conditions formulated at the low point of the chain O : $y(0) = 0$, $y'(0) = 0$.

From condition $y'(0) = 0$ it follows that $C = 0$ and from $y(0) = 0$ we have

$$0 = \frac{H}{w}\cosh 0 + D \, ,$$

$$D = -\frac{H}{w} \, .$$

This means that the shape of the chain can be formulated as

$$y = \frac{H}{w}[\cosh \frac{w}{H}x - 1] \, .$$

It should be noted that this curve is are often used in the construction of kilns and gateway arches (see e.g., Gateway Arch in St Luis or Gaudí's Casa Milá in Barcelona, Spain).

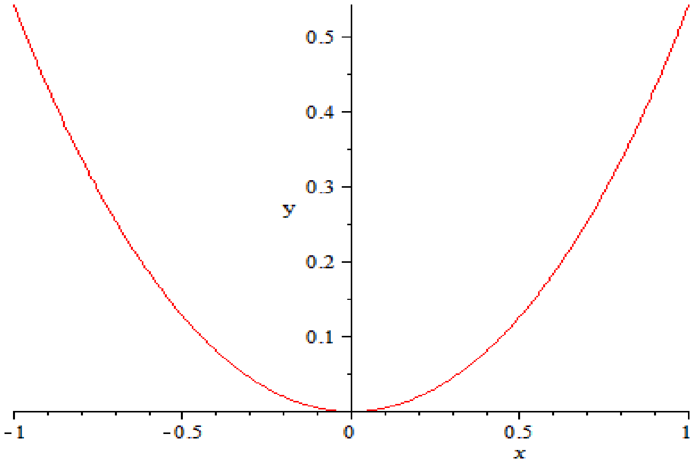

Figure 2. The figure represents the shape of the chain, the catenary

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