KINEMATIC ANALYSIS OF ROBOT AND MANIPULATOR ARMS

ATTILA OLASZ[1], TAMÁS SZABÓ[2]

Abstract: Robots and manipulators are used to serve machine tools in automatic production systems. The inverse kinematic analyses are carried out by Denavit-Hartenberg parameters for a 4 DOF manipulator and a 6 DOF robot. The aim of the paper is to determine the angles of the joints of the robot and the manipulator for the control system. Using trigonometric functions of triangles and the cosine theorem, the functions of joint angles are derived in closed forms. A computer code has been developed to simulate the prescribed motions of the robot and the manipulator. The same paths are prescribed for both the manipulator and the robot in numerical examples. The joint angles are calculated for a work cycle. The computed curves are similar but not the same for the first three joints of the robot and manipulator.

Keywords: robot, manipulator, inverse kinematics, joint angles

1. Introduction

The Department of Machine Tools at the University of Miskolc has built a hydraulic four degrees of freedom manipulator [3]. Such manipulators are used to serve different machine tools in industrial production systems. The Robert Bosch Department of Mechatronics has a FANUC LR Mate 200iC industrial robot with 6 DOF.

This paper deals with the analytic solution of inverse kinematics. The Denavit-Hartenberg parameters are used for the kinematic description [1], [2]. The first three joints of the manipulator and the robot have similar functions, i.e., they determine the position of the end effector. The orientation of the end effector can be given by the rest of the joints [2]. Therefore a robot can perform arbitrary orientation, but the manipulator provides only restricted possibilities.

The inverse kinematic problem of the manipulator can be solved analytically [2]. The object of this paper is to provide the reference inputs for the control of the hydraulic manipulator and the robot. The input parameters are the rotation angles of the joints. The motion of the end effector can be given by straight lines and arcs.

Due to lack of space, only the inverse kinematics of the manipulator is discussed in detail. The angles of the joints are given by trigonometric functions.

2. Geometry and kinematics of the robot and the manipulator

The simplified geometry of the manipulator and the robot is shown in *Fig. 1* and *Fig. 2*, respectively. Both the lengths of the members and the orientation of the revolute joints are

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shown in the basic position. The simplified models given in *Fig. 3* and *Fig. 4* are used for the kinematic analysis of the two mechanisms.

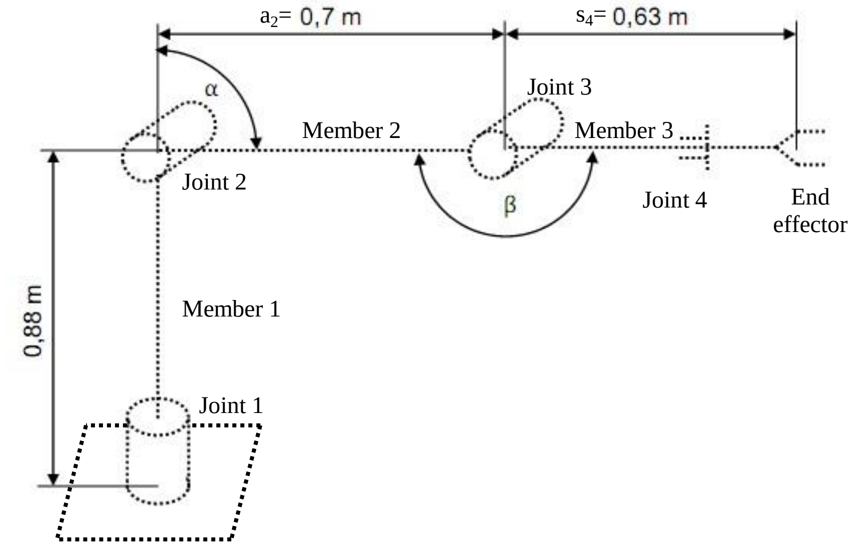

Fig. 1. Geometry of the manipulator

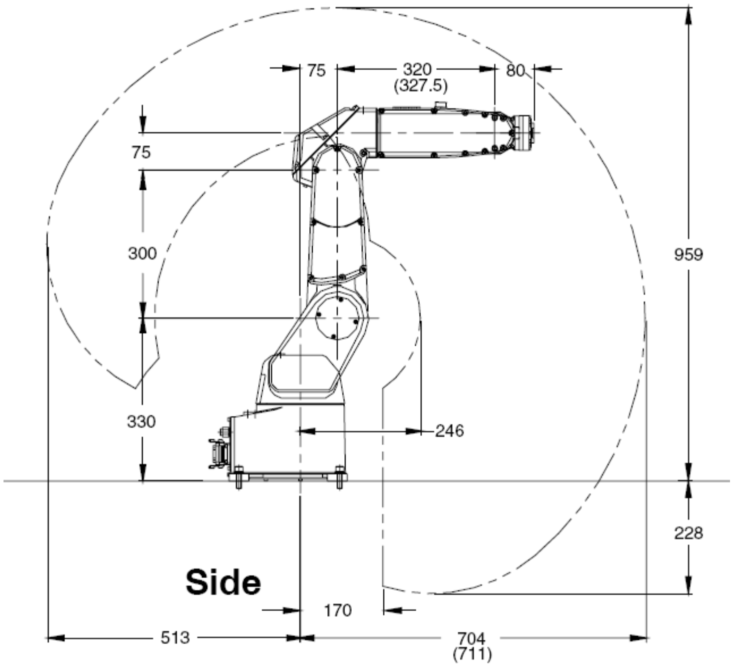

Fig. 2. Geometry of the robot

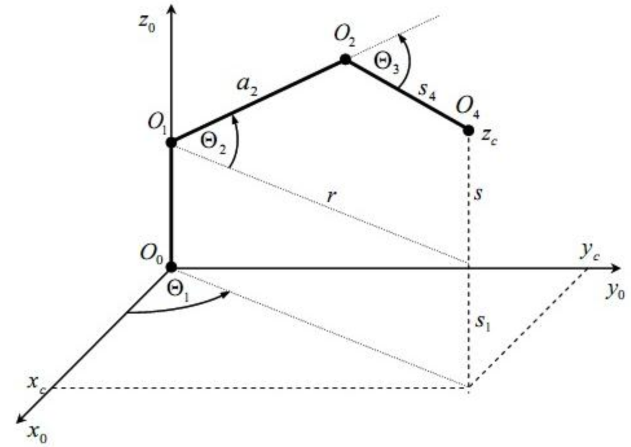

Fig. 3. Simplified model of the manipulator

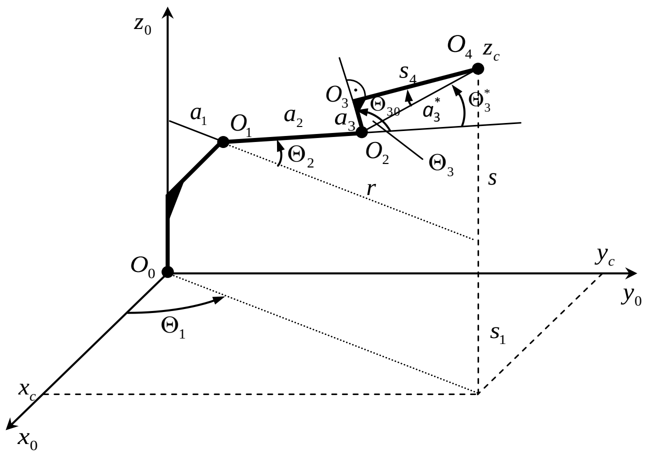

Fig. 4. Simplified model of the robot

Using trigonometric functions of the triangles in *Fig. 3* and *Fig. 4*, the angles of the joints can be given in closed forms both for the manipulator and the robot (*Table I* and *Table II*).

Table I. Joint angles of the manipulator

Angle	**Formula**
Θ_1	$a\tan 2(y_c, x_c)$
Θ_2	$a\tan 2\left(z_c - s_1, \sqrt{x_c^2 + y_c^2}\right) - a\tan 2(s_4 \sin\Theta_3, a_2 + s_4 \cos\Theta_3)$
Θ_3	$a\tan 2\left(\pm\sqrt{1-D^2}, D\right)$, where $D = \dfrac{r^2 + s^2 - a_2^2 - s_4^2}{2a_2 s_4}$

Table II. Joint angles of the robot

Angle	**Formula**
Θ_1	$a\tan 2(y_c, x_c)$
Θ_2	$a\tan 2\left(z_c - s_1, \sqrt{x_c^2 + y_c^2} - a_1\right) - a\tan 2\left(a_3^* \sin\Theta_3^*, a_2 + a_3^* \cos\Theta_3^*\right)$
Θ_3	$\Theta_3^* - \Theta_{30} + \frac{\pi}{2}$, where $\Theta_3^* = a\tan 2\left(\pm\sqrt{1-D^2}, D\right)$, $\Theta_{30} = a\tan 2(a_3, s_4)$
Θ_4	$a\tan 2(r_{12}s_1 - c_1 r_{22}, r_{22}s_1c_{23} + r_{12}c_1c_{23} + r_{32}s_{23})$
Θ_5	$a\tan 2\left(\pm\sqrt{1-(-r_{32}c_{23} + r_{22}s_1s_{23} + r_{12}c_1s_{23})^2}, -r_{32}c_{23} + r_{22}s_1s_{23} + r_{12}c_1s_{23}\right)$
Θ_6	$a\tan 2(-r_{33}c_{23} + r_{23}s_1s_{23} + r_{13}c_1s_{23}, -r_{31}c_{23} + r_{21}s_1s_{23} + r_{11}c_1s_{23})$

The last three angles of the robot depends on the orientation of the workpiece. The orientation of the end effector for the robot is given by the transformation matrix using Euler angles. The following notations will be used for the trigonometric functions in the sequel: $\cos\Theta_i = c_i$, $\cos(\Theta_i + \Theta_j) = c_{ij}$, $\sin\Theta_i = s_i$ and $\sin(\Theta_i + \Theta_j) = s_{ij}$.

The transformation matrices $\underline{\underline{H}}_{03}, \underline{\underline{H}}_{36}$ between the base and the third member and between the third member and the end effector are obtained by multiplications of the appropriate homogeneous transformations:

$$\underline{\underline{H}}_{03} = \underline{\underline{H}}_{01}\underline{\underline{H}}_{12}\underline{\underline{H}}_{23} = \begin{bmatrix} \underline{\underline{h}}_{03} & \underline{r} \\ \underline{0}^T & 1 \end{bmatrix}, \qquad \underline{\underline{H}}_{36} = \underline{\underline{H}}_{34}\underline{\underline{H}}_{45}\underline{\underline{H}}_{56} = \begin{bmatrix} \underline{\underline{h}}_{36} & \underline{r} \\ \underline{0}^T & 1 \end{bmatrix}$$

where homogeneous transformation $\underline{\underline{H}}_{i,i+1}$ is given by the so-called Denavit-Hartenberg parameters, and $\underline{\underline{h}}_{03}$, $\underline{\underline{h}}_{36}$ are orientation matrices which are detailed as:

$$\underline{\underline{h}}_{03} = \begin{bmatrix} c_1(c_2c_3 - s_2s_3) & s_1 & c_1(c_2s_3 + c_3s_2) \\ s_1(c_2c_3 - s_2s_3) & -c_1 & s_1(c_2s_3 + c_3s_2) \\ c_2s_3 + c_3s_2 & 0 & s_2s_3 - c_2c_3 \end{bmatrix}, \ \underline{\underline{h}}_{36} = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & c_4s_5 & c_4c_5s_6 + c_6s_4 \\ c_4s_6 + c_5c_6s_4 & s_4s_5 & c_5s_4s_6 - c_4c_6 \\ -c_6s_5 & c_5 & -s_5s_6 \end{bmatrix}.$$

The remaining three angles can be determined using the following equality

$$\underline{\underline{h}}_{36} = \left(\underline{\underline{h}}_{03}\right)^T \underline{\underline{h}}_{06},$$

where $\underline{\underline{h}}_{06} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ is prescribed by the user.

3. Numerical example

A logistic problem of an automatic production system is shown in *Fig. 5*. The same material flow denoted by the dashed line, i.e., the motion of the workpiece is carried out by a manipulator or a robot.

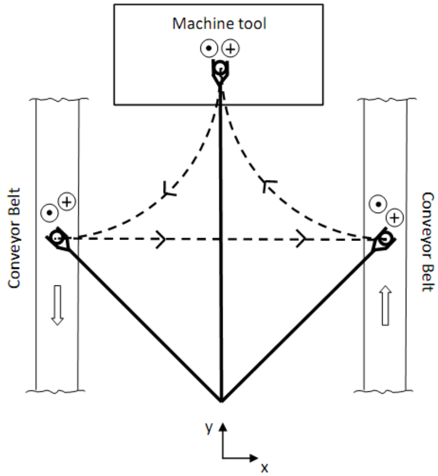

Fig. 5. A logistic problem of the manipulator and the robot

The angles of the joints and the motion of the manipulator and the robot are computed by a code based on the theory detailed in Section 2. The simulated motions are given in *Fig. 6* and the angles of the joints are shown in *Fig. 7* for the manipulator and the robot, respectively. The numbers in *Fig. 7* denote the relevant joints.

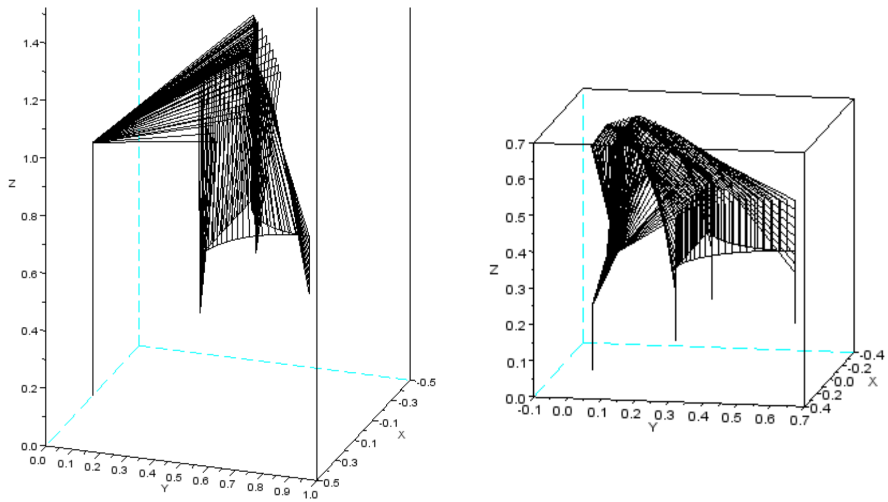

Fig. 6. Motions of the manipulator and robot

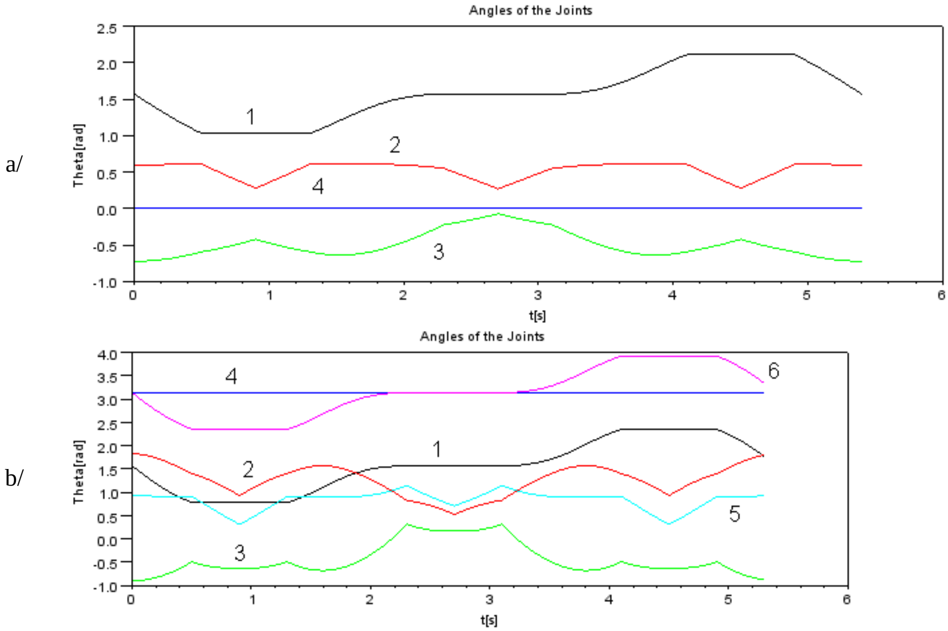

Fig. 7. Agles of the joints of the manipulator (a) and the robot (b)

4. Summary

The analytical formulae of joint angles have been derived both for a manipulator and a robot. A computer code has been developed to determine the reference angle inputs for the joints of the manipulator and the robot. When the joint angles are known, the control system can perform the prescribed motions of the manipulator and the robot. The same paths were prescribed for both the robot and manipulator. The computed angles of joints are similar for the first three joint but not the same since the geometries of the robot and manipulator were different. Comparing a 6 DOF robot and a 4 DOF manipulator, the previous one can be used more flexible, since one can prescribe not only the position of an workpiece but also its orientation.

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