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APPLICATION OF THE QUEUING THEORY TO MODELLING THE STRUCTURE OF THE WORKSHOP IN THE SPHERE OF ORGANIZING THE OPERATION OF THE COMPANY

TADEUSZ GŁOWACKI[1]

Abstract: The theory of queues is a field of operational research within the scope of the theory of stochastic processes. The primary objective of this theory is to develop methods that allow determination of the basic indicators characterizing the process of handling and evaluation of the work quality of the queuing system and the choice of optimal structure and organization of the service. The purpose of this article is to show the application of the queuing theory to modelling the selected structure of the administration of the internal orders flows for the performance of service by the mechanical workshop for the maintenance of the company.
Keywords: logistic, queuing theory, staff management.

1. Introduction

Initially, the theory of queues was closely associated with technology, but due to its effectiveness and flexibility it has been used in many other areas. The precursor of the queuing theory was the Danish teletechnik, A.K. Elring, who in 1909 published a paper on the telephone exchange model [1, 3, 4]. Issues of queuing systems and networks can be successfully applied to the modelling of service systems, operating in many areas of economic life and to develop methods for the overall characteristics of the process [1, 5, 4]. The result of the application of this theory may be more optimal decisions regarding the structure, organization and operation from different stakeholders' point of view. In the case of least efficient systemic structures - queuing networks allow identification of bottlenecks. Consequently, it is possible to optimize these structures taking the costs of operation and its maintenance into account. Thanks to the use of queuing network models different options can be compared and the best solution can be used in practice.

2. The essence of queuing networks

The model that will be presented is Jackson's two-class queuing system with unlimited queue notifications and unlimited waiting time [2, 4, 7]. Such a system may be in states with no queue, if the number of notifications is less than or equal to the number of service channels. Otherwise, the system is in the states with the queue.

The process consists of the service station for those who work according to the FIFO (First In First Out) discipline. Entering external notifications shall be subject to the Poisson distribution with parameter - λ -. Each service station may comprise one or more channels using a coefficient - μ- which may depend only on the number of entries in a station, which is identical for all channels of operation. Entries operate in accordance with an exponential distribution. The relative intensity of use in the system - r - channel is described by dependency.

[1] Deputy Director of the technical issues, Wastewater Treatment Plant "Warta" JSC in Czestochowa
glowacki_tadeusz@poczta.fm
ul. Srebrna 172/188; 42-201 Częstochowa, Poland

The system is stable if the following condition is satisfied:

$$\lambda > r \cdot \mu, \quad p = \lambda / (r \cdot \mu) \tag{1}$$

where

λ - the arrival rate - the average number of orders per unit of time,
μ - operating rate - the average number of orders handled per unit of time,
r - number of parallel operating channels,
p - traffic intensity parameter - the ratio of the number of incoming orders to the number of orders handled per unit of time.

Only situations in which the orders are handled as they arrive to the point of implementation shall be considered all orders are treated equally.

The probability of a steady-state is expressed by a formula:

$$P(n) = \begin{cases} \frac{p^n P(n=0)}{n!}, & n < r \\ \frac{p^{r-n} P(n=0)}{r!}, & n \geq r \end{cases} \tag{2}$$

Probability that there is a lack of orders in the system n=0:

$$P(n) = \frac{1}{\sum_{i=0}^{i=r-1} \frac{p^i}{i!} + \frac{p^r}{(r-p)(r-1)!}} \tag{3}$$

Probability that there are more than n> 0 orders in a queue dla $n_0 \geq r-1$:

$$P(n > n_0) = \frac{r^{r-n_0} p^{n_0+1} P(n=0)}{(r-p)r!} \tag{4}$$

Average number of orders waiting to be accepted:

$$Q = \frac{p^{r+1} P(n=0)}{(r-p)^2 (r-p)!} \tag{5}$$

While analyzing queuing networks, we consider two cases:

1) when the system tends to equilibrium, it is expressed as $\lambda < \mu$ rx if both values are constant, then the probability that the queue has the same length is constant at each time unit,
2) when $\lambda > r \cdot \mu$ the system is unstable and the probability of a long queue grows.

3. Operationalization of the queuing theory

We evaluated service stations of technological devices for the purposes of maintenance of the company. Operation is normally done by a mechanical workshop with three positions. The analysis covered a period of four consecutive months at the end of the second and third quarter of 2014. The aim of this study was to determine whether the order handling system tends to equilibrium.

Handling orders in the audited company is based on the model of an open one-class network queue built from Markowski's systems where times of handling orders are random variables with exponential distribution. The system has three operating positions. The scheme of this system is represented by Figure 1.

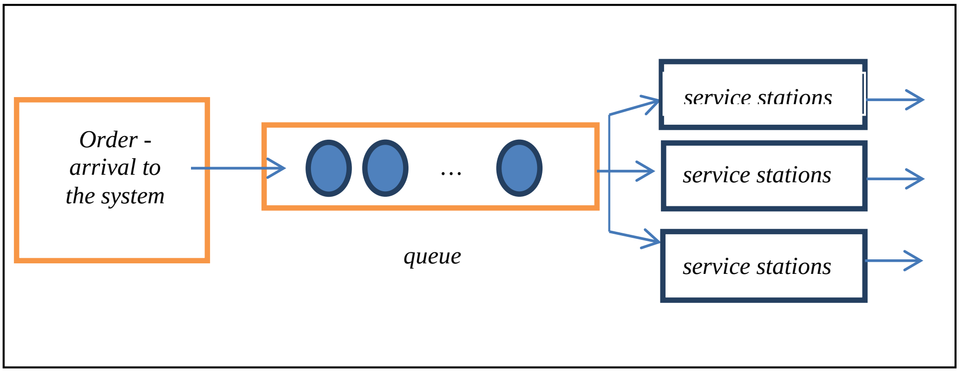

Figure 1. Example of schema queuing system

Basic variables characterizing the test system are given in Table I.

Table I.

The basic variables defining the investigated system

Month	Number of orders	Total time of order arrival	The rate of arrival λ	Average realization time [h]	μ x r	The relative intensity of service -p-	stability of the system	Volume overload [%]
I	68	136	0,50	6,58	0,456	1,096	unstability p>1	9,6
II	75	160	0,47	6,76	0,444	1,058	unstability p>1	5,8
III	84	160	0,52	6,15	0,489	1,063	unstability p>1	6,3
IV	96	184	0,52	6,00	0,501	1,019	unstability p>1	1,9
$\bar{X}$	80	160	0,502	6,37				5,9

Since the value of λ > μ · r, we have to deal with an unstable system, that is, the arrival rate exceeds the rate of implementation and the probability of a long line increases because the system cannot make up for time for the execution of tasks. Therefore, a solution that makes the system stable should be proposed. Achieving equilibrium is possible only by taking measures that have reduced order service time or providing an additional service station. Since the degree of volatility or overload volume is low and averages 5.9% in the investigated period, even the reduction of the number of serviced orders can be considered. Then, a surplus of orders should be outsourced.

To adopt appropriate solution for the problem, two options should be considered:

1) How many orders can a Workshop realise to reach a state of equilibrium?
2) How installing the additional service stations will affect parameters of the system? For those variants, the probability of the steady state was calculated. The results are shown in Table II.

Table II.

Limiting number of possible orders service (change of λ)

Month	**Number of orders**	**Total time of order arrival**	**The rate of arrival λ**	**The traffic intensity p' = λ -/ (μ·r)**	**μ · r**	**λ'**	**The number of possible orders**	**Number of orders to outsource**
I	68	136	0,50	1,00	0,456	0,456	62	6
II	75	160	0,47	1,00	0,444	0,444	71	4
III	84	160	0,52	1,00	0,489	0,489	78	6
IV	96	184	0,52	1,00	0,501	0,501	92	4
średnio	80	160	0,502	1,00	0,472	0,472	76	5

Stabilizing the system by reducing on average 5 orders executed in their own system per month. This a rational choice which is not a financial burden while increasing the amount of stations. Also, when comparing the probability of the steady-state for a variant 1 it can be considered as fully acceptable from the standpoint of system operation.

After modification, a queuing system will look like the model in the Figure 2.

Table III.

Change of intensity of service (r = 4)

Month	Number of orders	Total time of order arrival	The rate of arrival λ	Average realization time [h]	The rate of service μ	μ · r'	The relative intensity of service -p-	Stability of the system	The degree of under load [%]
I	68	136	0,50	6,58	0,152	0,608	0,822	stability p<1	18
II	75	160	0,47	6,76	0,148	0,592	0,794	stability p<1	21
III	84	160	0,52	6,15	0,163	0,652	0,797	stability p<1	20
IV	96	184	0,52	6,00	0,167	0,704	0,738	stability p<1	30
$\bar{x}$	80	160	0,502	6,37	0,157	0,639	0,787	stability	21

Table IV.

The probability of a steady state for the mean values

Characteristic quantity	variant 1 table 2	variant 2 table 3
The probability that there will not be any queue	36%	45%
The probability that there are more than two orders in a queue	9%	4.5 %
Average of orders waiting in the queue for acceptance	0,045	0,00

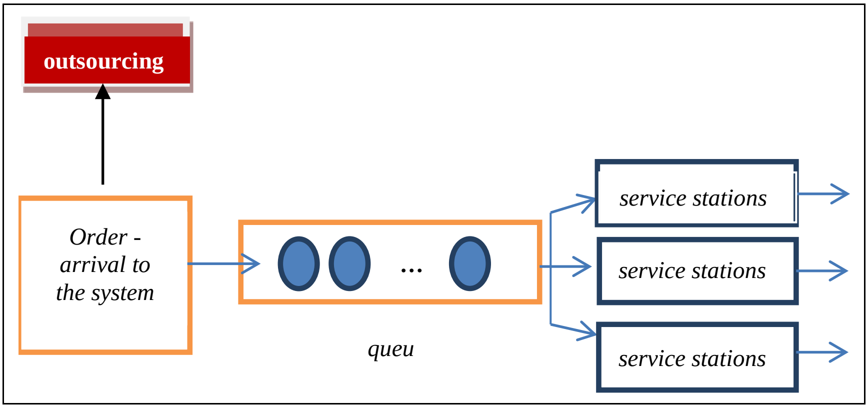

Figure 2. Modified queuing system

4. Summary

During the analysis, the principle of Jackson's networks became evident. Owing to the above analysis, the network with the three positions could be modified. After a detailed analysis of simulation results and calculations, the conclusion is that when the number of orders increases, we can expect a growing queue. This situation is unacceptable in terms of an industrial enterprise systems maintenance. In contrast, when the number of service stations is large, we see that they are not used, which is an uneconomical operation. The situation from the point of view of managers has become comfortable in the newly adopted configuration. Although the probable lack of queuing acceptance is high, over 54%, the probability of waiting in the queue for more than 2 orders is small and amounts to approx. 9%, and average queue expects only an average of approx. 0.04 order. Therefore, it can be assumed that the theory of queues is a good tool for modelling the structure of the workshop in the organization of operating the running-a-company maintenance and allows for the proper design of queuing service system for internal orders in the company.

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