

## **THE EFFECT OF THE SUPPLY ACCURACY AND THE DEMAND-CHANGES ON THE INVENTORIES AND ON THE COSTS**

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**Abstract:** The frequency of shortage occurrence depends on the accidental impact of several factors, so the occurrence of a stock shortage is an event that cannot be planned with full certainty. This means that the factors causing the shortage and the occurrence of the expected shortage can be defined only with random variables. By detailing the connection between the fluctuation of demands and the safety stocks, we will present the impact of quantitative factors, and we will analyze the connection between supply accuracy and safety stocks along the time factors. By walking around the periodic and continuous stock review models during our analysis, we will present the relations between the safety stocks and the fluctuation of supply accuracy and also the effect of the demand-changes. We will define the formula for the quantification of the safety stock as well as the occurring costs in connection with stock management and stock shortage.

**Keywords:** change of demands; shortage, standard normal distribution; random variable, stochastic, cost

### **1. Introduction**

All production companies require for operation the acquisition of items from external resources, and their arrival by the specified deadline. When the stock replenishment can be exactly on time realized, the level of the closing stock, the quantity of the placed order and the actual date of the order can be clearly defined with the knowledge of the initial stocks, the production demand and the restocking of supplies. However, this initial condition is very rare in practice. There are several unpredictable factors influencing the supply accuracy that will affect the operation of the production company. The objective of the logistics management is to guarantee the stock level required for the adequate handling of production at the lowest possible level of costs [1].

The importance of supply accuracy on the inventories and supply chain efficiency is growing. The operation research and the management sciences literature includes a huge number of methods and models to support the design, optimisation and control of purchasing processes from the point of view supply accuracy [2].

The integration is a core problem of supply in the field of logistics, because the processes of purchasing and production have bidirectional effects. The integration of the first two functional parts of logistics (purchasing, production) is especially important in the case of multistage supply [3].

The supply chain problem includes a wide area of decisions: supplier selection [4], payment management of supply processes [5], coordination of multi-level, vendor managed inventory systems [6], consignment stock management [7], risk management of supply chain and stock management [8]. The modelling of supply processes can be described from

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the point of view analysis and optimization of warehouses, and there are important sources proposing approach to reduce inefficiencies using value stream mapping [9]. The supply chain accuracy is influenced by the size, operation and scheduling of fleet [10]. It means, the control of the supply processes includes external and internal factors to be optimised. Within the frame of this paper authors are focusing on the supply accuracy on the inventories.

## 2. The stochastic time and quantitative factors as random variable

In practice, several unexpected events can lead to a shortage of stocks, e.g. excess usage due to a reject occurred for any reason within the production process, stock deviation, late delivery due traffic obstacles, stocks with inappropriate quantity or quality etc. These factors can be classified along two main groups. The first group of factors includes uncertainties related to time, events the occurrence of which affect the availability of stocks in time. The other group of factors includes quality-related impacts, the occurrence of which causes a stock level modification of an unanticipated extent.

The impacts in both factor groups can have two directions. This means that in the case of the time factor, supplies ahead of the schedule can happen along with delays in delivery, and in the case of quantitative factors, along with an unexpected excess usage, the stock discovered during an inventory can lead to the increase of the stock level. Accordingly, the challenges for some of the impacts are the management of the stock shortage and its consequence, and for impacts with the opposite direction, the challenges are the consequences related to the increased stock level.

Along the two groups of factors, we present the relations related to the avoidance of stock shortage, and the method of decreasing the occurrence level of the stock shortage to an acceptable level.

**2.1. The relationship between the deterministic changes of demands, delivery performance and the stock level.** As a simplified model, Figure 1 shows the inventory mechanism and the modification of stocks in case of a deterministic demand and stock replenishment.

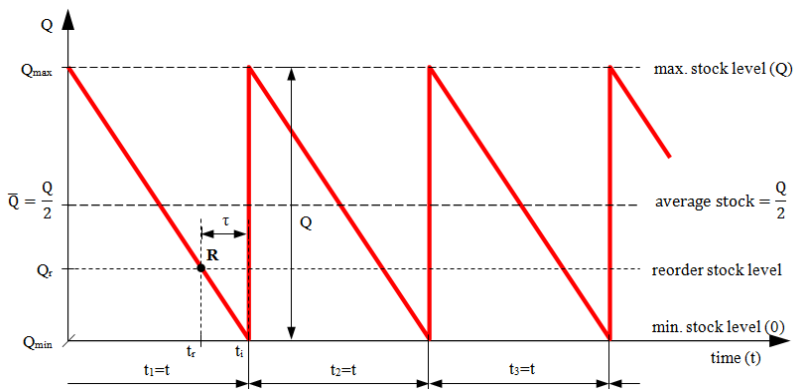


Figure 1. Development of stock level in case of deterministic demand and replenishment time

The initial conditions of the model include the continuous use at a steady pace, a steady  $t$  periodic supply, the minimum stock level  $Q_{min}$  equal to zero starting from the inadmissible stock shortage and the zero safety stock, and the  $Q$  reorder quantity until the admissible  $Q_{max}$  maximum stock level. The calculable  $\tau$  replenishment time and the  $t_i$  date of receipt determine the latest  $t_r$  date, when the order must be placed [11]. The  $Q_r$  stock level assigned to time  $t_r$  represents the minimum stock level that covers the usage within the time period necessary from the placement of the order until the actual receipt of the ordered quantity [12] [13].

**2.2. Continuous stock review in case of stochastic replenishment period.** By using the principle of *ceteris paribus*, we carry out the analysis by emphasizing one initial condition of the model, the development of the arrival time of purchased parts. In this case, the model changes to the extent that we allow a deviation in any direction and of any extent between the lengths of the different periods, by specifying that the demand of the respective period is continuous and constant within the period (almost stationary) [11].

The simplified stock management model showed on Figure 1 and characterized with a deterministic stock replenishment time and a utilization with constant intensity can be completed with the maximum extent of the deviation of supply accuracy in two directions, so that Figure 2 is modelling the deviations occurring compared to the planned utilization. Since the direction and extent of the deviation compared to the plan is stochastic, i.e. it cannot be exactly defined in advance, thus the development of the actual arrival date and of stocks can be determined only with random variables that are represented on the Figure 2 by the probability density functions of the standard normal distribution.

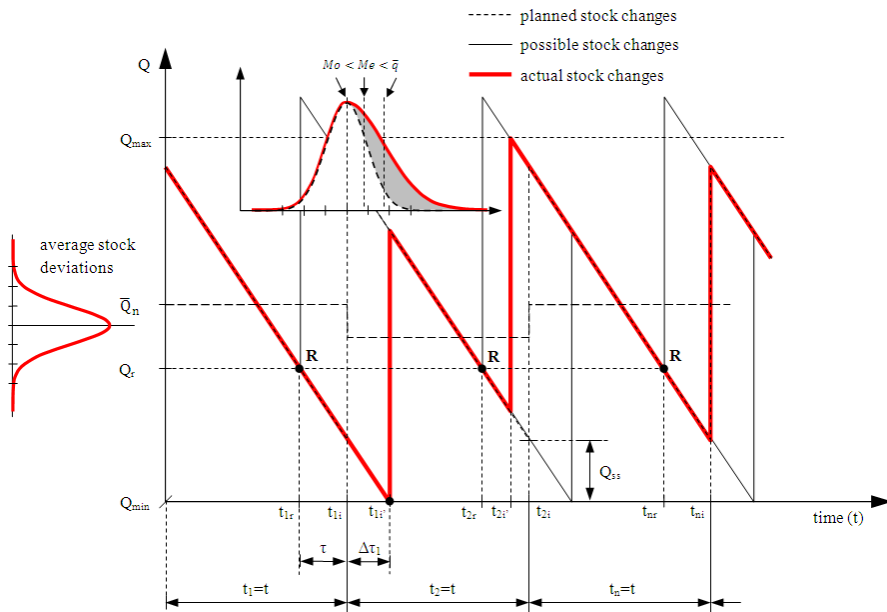


Figure 2. Continuous stock review in case of stochastic replenishment period

Since the fluctuation of supply accuracy has a stochastic character, the possible results are random variables equivalent to the standard normal distribution.

In case we only keep stocks to cover the demands of the planned length of the period, according to our stock management strategy, the delayed receiving of the purchased parts compared to the planned  $t_i$  arrival date would result in a drop to zero of the stocks before the  $t_i$  date of receipt. This is unacceptable in case we start from a strategy that does not allow a stock shortage, a certain amount of safety stock ( $Q_{ss}$ ) should be kept in order to avoid the risks and costs due to delayed deliveries.

**2.3. Periodic review in case of stochastic change of demands.** In the next model we carry out the analysis by emphasizing one initial condition of the model, the development of the utilization demand. In this case, the model changes to the extent that we allow a deviation in any direction and of any extent between the demands of the different periods, by specifying that the demand of the respective period is continuous and constant within the period (almost stationary) [11].

The simplified stock management model showed on Figure 1 and characterized with a deterministic stock replenishment time and a utilization with constant intensity can be completed with the maximum extent of the demand change in two directions, so that Figure 3 is modelling the deviations occurring compared to the planned utilization. Since the direction and extent of the deviation compared to the plan is stochastic, i.e. it cannot be exactly defined in advance, thus the development of the actual utilization demand and of stocks can be determined only with random variables that are represented on the Figure 3 by the probability density functions of the standard normal distribution. Since the fluctuation of demands has a stochastic character, the possible results are random variables equivalent to the standard normal distribution.

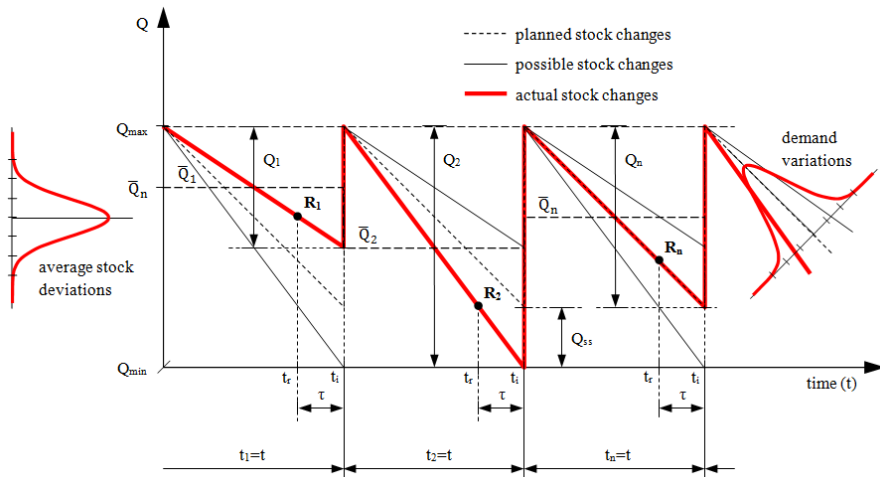


Figure 3. Periodic review in case of stochastic change of demands

In case we only keep stocks to cover the planned demands, according to our stock management strategy, the increase of the actual utilization demand compared to the plan

would result in a drop to zero of the stocks before the  $t_i$  date of receipt. This is unacceptable in case we start from a strategy that does not allow a stock shortage, a certain amount of safety stock ( $Q_{ss}$ ) should be kept in order to avoid the risks and costs due to excess consumption (Figure 4.).

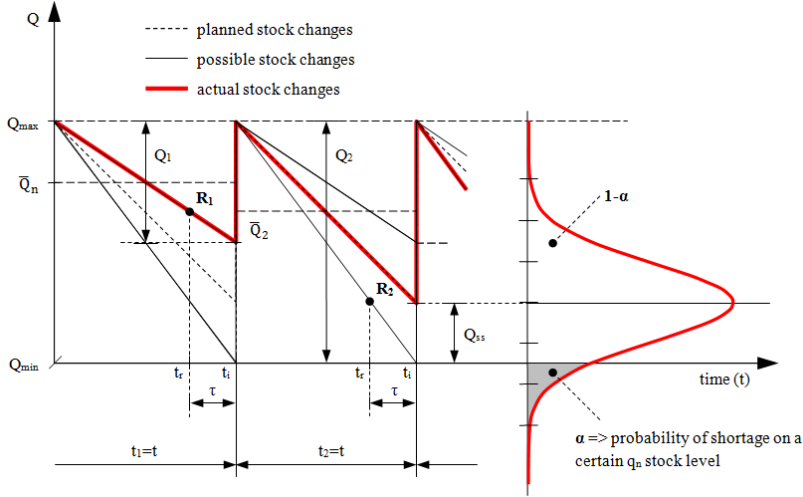


Figure 4. The relations of the periodic review

Since the expected value of the utilization demand and the fluctuation of utilization are random variables due to the impact of future events with unpredictable outcomes, the development of the  $Q_{max}$  stock level will be a random variable as well. The probability of a maximum stock level of a given period to be equal to the planned  $Q_{max}$  stock level, is equal to the difference of the  $F(q) - F(\mu)$  probabilities, i.e.  $P(q = \mu)$ .

### 3. The effect of the safety stock on the occurrence probability of the stock shortage

The density function detailed on Figure 5 shows the probability of occurrence of actual quantities, which reflects that the frequency of occurrences is more typical around the expected value, while their probability decreases towards the two ends of the function [14] [15] [16] [17].

The Figure 5 shows clearly that the extent of the given stock level covering the demand can be defined in relation to any arbitrary  $q_n$  stock level and the actual demand. The stock level above the actual demand is the safety that covers the fluctuation of demands. The safety stock level can be calculated with the following formula:

$$Q_{ss} = q_n - \mu \tag{1}$$

where

$Q_{ss}$  – safety stock level,

$\mu$  – actual demand, or the expected demand value,

$q_n$  – arbitrary stock level.

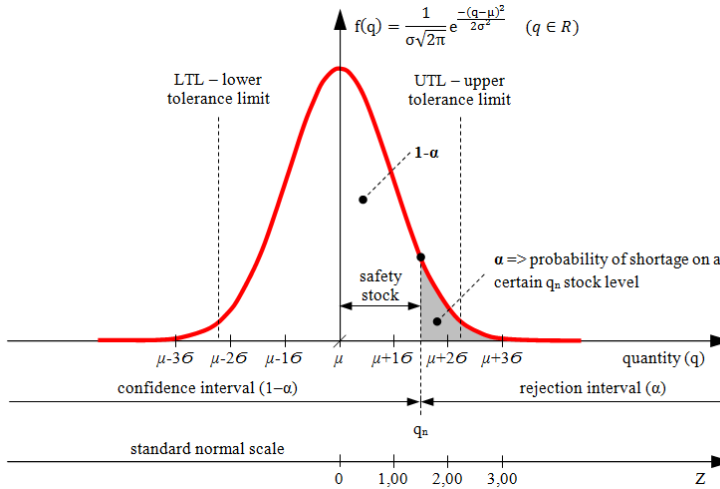


Figure 5. The effect of the safety stock on the occurrence probability of the stock shortage

The formula above demonstrates that the safety stock can have a negative level, if we allow the probability of the occurrence of a stock shortage during our stock management strategy to reach an extent where the stock we keep in certain periods is not enough to cover the planned demands:

$$q_n < \mu \quad \text{in case of} \quad Q_{ss} < 0 \quad (2)$$

In the knowledge of the standard deviation and the expected value, the probability of the occurrence of a shortage can be defined in relation to any utilization demand and stock level. In case we adjust the stocks only to the planned demand level, i.e. we do not keep a safety stock ( $q_n = \mu$ ), the probability of the occurrence of a shortage in case of a demand with standard normal distribution will correspond to the probability of the non-occurrence of the shortage (Figure 6). Since the combined probability of the two possible results is 100%, the following relation is valid:

$$P_{(shortage)} = P_{(coverage)} = 0,5 \quad (3)$$

The distribution function on Figure 6 shows the probability of the coverage of demand fluctuations in relation to the changes of the stock level. The probability of the occurrence of the shortage decreases proportionally with the increase of the safety stocks; however, the complete safety can be guaranteed only by a stock with an infinite level. The introduction of the distribution function is justified by the fact that the function reflects well the probability of the occurrence of the stock shortage in connection with the stock level. The probability can be expressed as a risk as well, to which actual costs can be assigned.

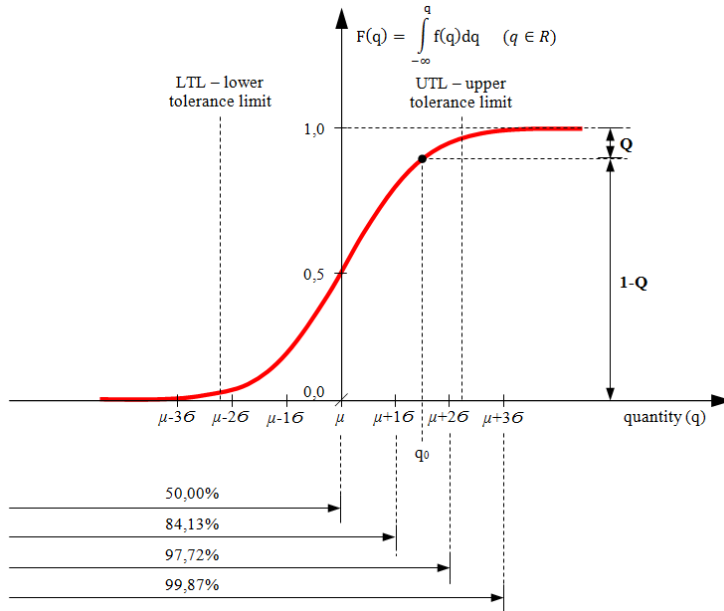


Figure 6. The probability of the coverage of demand fluctuations in relation to different stock level

#### 4. The relation of stock shortage and stock management costs as a function of the stock level

The relations shown in connection with the stock level can be applied during the definition of costs as well. During the modelling of costs arising in connection with the avoidance of stock shortage, we do not calculate with the purchase costs in order to simplify the process. This is also justified by the fact that the use of purchase costs and stock management costs was already necessary for the definition of the steady periods and the economically efficient order quantity defined on Figure 1 as initial conditions. Another reason to ignore the purchase costs is that within the model we assume that every demand arising during the whole period and the fluctuations are covered with the stock, however, the total quantity of the demands does not change, and there will be deviations only in the extent of demands of the different periods due to fluctuations. Accordingly, a smaller stock must be reordered in one period due to a smaller demand, and a bigger stock in another period. Similarly, the purchase costs fluctuate as well, but these fluctuations will level out within the analyzed period.

During the analysis of costs, we will quantify only the relations between the stock management and the stock shortages. The density function on Figure 7 and the inverse distribution function on Figure 8 show the phase of stock management, where the stock levels can reach a higher or similar level than the expected value of demands. Figure 7 shows that in case the stock level has a value higher by one standard deviation than the value of the demand, the probability of the stock shortage decreases by 34.1%. As a relation, we can state that starting from the symmetric functions, the fluctuation of the demands as defined in relation (3) reflect a smaller utilization in case of 50% of the outputs

compared to the plan, thus a stock level higher by 1 standard deviation than the expected value of demands decreases the probability of the stock shortage's occurrence by 84.1%.

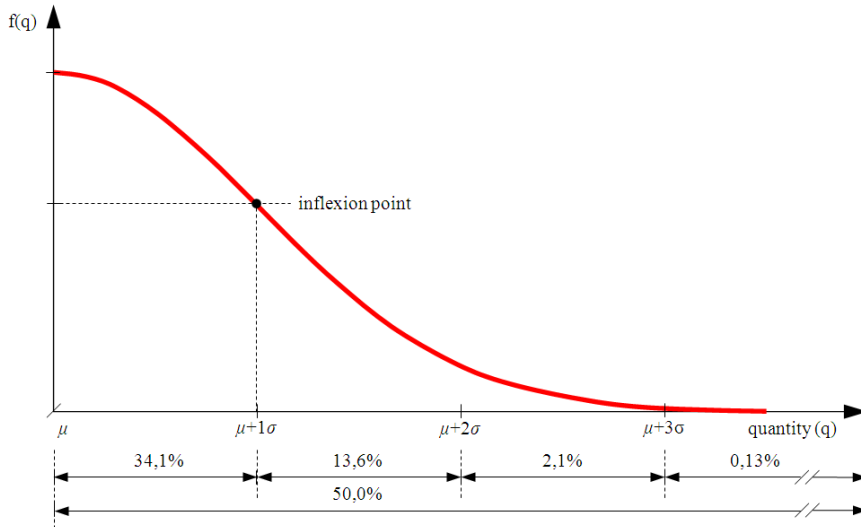


Figure 7. Probability of stock shortage in case of a stock that exceeds the level of expected demand

The probability of the stock shortage assignable to the different stock levels can be shown on a continuous monotonically decreasing inverse distribution function (Figure 8). The higher the stock level, the lower the probability of a shortage and thus the costs of stock shortage, and at the same time, the higher the costs related to stock management.

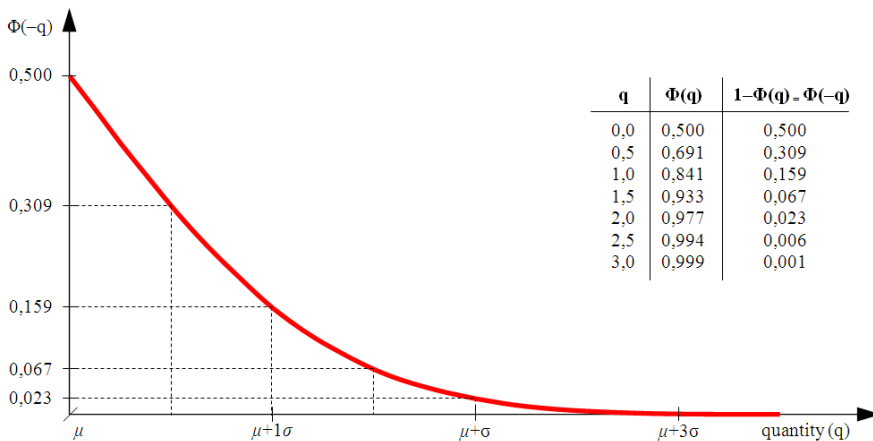


Figure 8. Probability of stock shortage in case of a stock that exceeds the level of expected demand

In order to quantify the optimum stock level within our stock management strategy, we can define the level of costs caused by a stock shortage with a certain extent. Different



probability values can be assigned to different stock levels, and these probability values can be represented with the help of standard normal distribution tables. In case the probability of the stock shortage is a random variable, the level of the assigned costs will be a random variable increased by a factor, and this factor will price the costs arising from the shortage's probability. Figure 9 shows the costs in relation to the different stock levels.

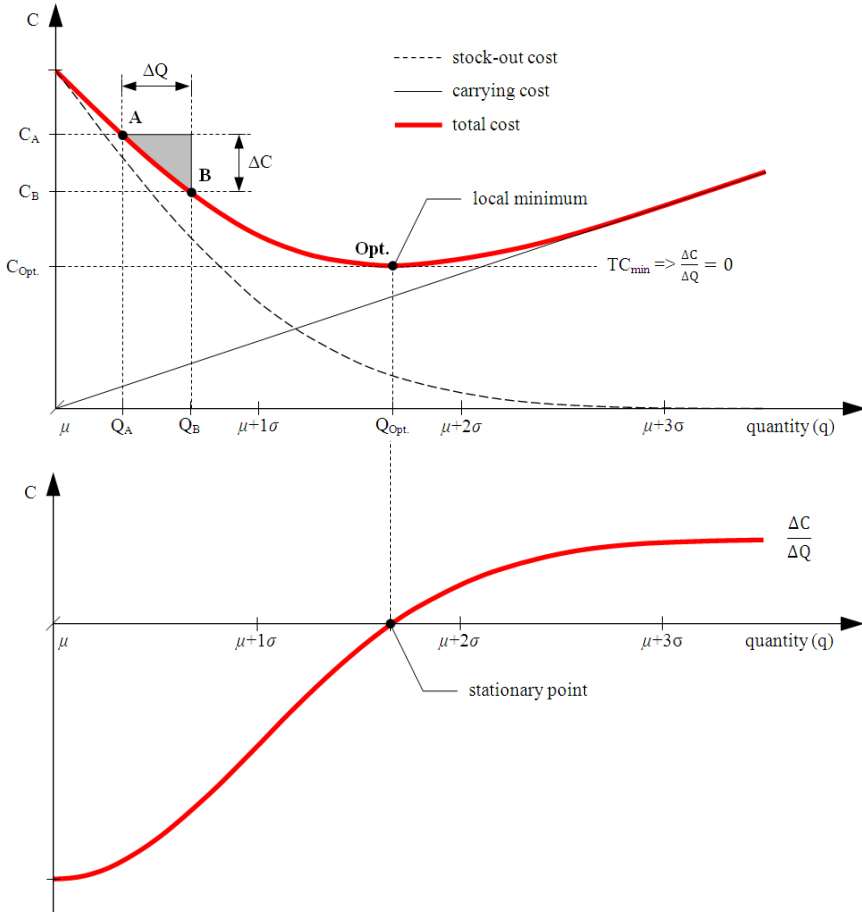


Figure 9. The relation of stock shortage and stock management costs as a function of the stock level

After drawing the costs of stock management onto the diagram, we can define the relative movement of the two costs. By assuming a cost-minimizing rational decision, a stock level should be targeted where the combined costs of stock shortage and stock management are minimal. When presenting the functions, it should be emphasized that since the costs of stock shortage were deduced from the values of the distribution function, the optimum point of the combined costs are not necessarily positioned at the intersection of the stock management and stock shortage functions.

The optimum point of the combined costs can be defined by calculating the extreme values after deriving the function. The stock level for the optimum combined costs is where the stationary point of the continuous function of a single variable is at the same time monotonically increasing.

To simplify the model, we indicated the costs of stock management with the line starting from the initial point. However, in practice, the stock level and the attributable holding costs do not equal zero at a 50%-value of the distribution function. He have to mention that the diagram reflects the safety stock created to decrease the uncertainties in the time factor and the quantitative factors, so it presents the development of the stock level that is necessary to avoid the occurrence of the shortage beyond the stock level assigned to the planned demand and the lead time. The connection on Figure 9 shows that the by a zero level of the safety stock, the average stock level can be quantified by normal operation, and the costs of stock management can be defined as well.

The following tasks must be performed when defining the stock level enabling a continuous operation by an optimal cost level:

- First, the service level of our stock management strategy has to be defined, i.e. the frequency of shortage acceptance
- The different cost elements must be priced, thus the development of the holding costs must be defined in relation to the stock level, and the consequence of the shortage occurrence must be quantified. The consequence of shortage cannot be defined with full certainty in practice, since a certain amount of uncertainty is always involved. The majority of processes within the organization, such as the minus production time due to a missing part, the readjustment of the production line for another product can be delimited. However, factors outside the organization, e.g. the deterioration of the company's judgement by the clients and the absence of future orders are completely unknown consequences.
- In the knowledge of the relations between cost elements and stock level, the function of the complete costs must be drawn up, and a minimum value must be set.
- The stock level assignable to the minimum value of total costs must be defined. We can see that the function of total costs has its minimum value at the point where the rise of the stock shortage function corresponds to the rise of the stock holding function. Accordingly, we can imagine the rare case in practice, where the costs of stock shortage have a lower rise in every phase of the function than the stock holding function, negligible stock shortage costs can occur in extreme cases, a fact that shows that the function of total costs has its minimum value by negative values of the safety stock. In practice, this means that the stock would not cover any part of the planned demand, thus from the point of view of costs it is more beneficial for the enterprise to allow the occurrence of a stock shortage to the detriment of the satisfaction of planned demands in some cases, since the consequence of the occurrence of stock shortage is smaller than the sacrifice of stock holding would constitute.
- We must define the extent of the expected probability of a stock shortage occurrence by an optimal safety stock level assignable to the minimum value of the total cost function.

- We must analyze the relation of the probability of the allowed stock shortage by the planned service level and the probability of occurrence of the expected stock shortage by the optimization of costs.
- In case these two probabilities are not identical, and the result reflects that the probability of the shortage by the optimization of costs is higher than the probability of the shortage with acceptable extent related to the foreseen service level, the necessary decision must be made to keep the planned service level despite of the increase of costs, that is, we optimize the costs by a lower service level.

## 5. Summary

The more stringent expectation we define towards the frequency of stock shortages in our stock management strategy, the higher the expenses will be for the avoidance of the stock shortage.

In case of stochastic demands we can guarantee the achievement of the target level in two directions:

- we can decrease the uncertainty of the demand fluctuation with a more accurate forecast.
- we can increase the safety stocks to cover the fluctuating demands.

In case of stochastic delivery performance we can work on the achievement of the target level by two methods:

- with a stricter supplier qualification, we can reduce the uncertainty of supplies,
- we can increase the safety stocks to cover the fluctuating arrival date of purchased parts.

The management of utilization demand changes and stock replenishment time changes of unpredictable extents urges the management of the production company to reach a compromise. The shortage of stocks can cause serious disturbances in the production supply and customer service, the costs of which are often unquantifiable. The avoidance of stock shortages is an important objective for every organization; however, it would be possible only with the management of an infinite stock level due to the stochastic nature of demands and delivery performance. Stock management also has costs, thus the definition of an optimum stock level is a task for an enterprise, where the combined costs of stock management and stock shortage show the lowest possible level.

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